

$$I \int_0^3 \frac{x \, dx}{\sqrt{x+1}}$$

let $u^2 = x+1$ $x=0 \Rightarrow u=1$
 $x=3 \Rightarrow u=2$
 $2u \, du = dx$

$$2 \int_1^2 \frac{(u^2-1)u \, du}{u}$$

improper
 evaluation of
 limit integral
 - 2 PTS

$$2 \int_1^2 (u^2-1) \, du$$

$$2 \left(\frac{u^3}{3} - u \right) \Big|_1^2$$

$$2 \left(\frac{8}{3} - 2 - \frac{1}{3} + 1 \right) = \frac{8}{3}$$

$$II \int_0^1 x \sin^{-1} x \, dx \quad \text{Parts.}$$

$u = \sin^{-1} x \quad dv = x \, dx$

$du = \frac{1}{\sqrt{1-x^2}} \, dx \quad v = \frac{x^2}{2}$

$$\frac{x^2}{2} \sin^{-1} x \Big|_0^1 - \frac{1}{2} \int_0^1 \frac{x^2 \, dx}{\sqrt{1-x^2}}$$

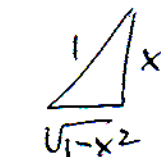
$$\frac{1}{2} \sin^4 \theta = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{\sin^2 \theta \cos \theta \, d\theta}{\cos \theta}$$

$$\frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{4}$$

$$-\frac{1}{2} \int_{\frac{\pi}{2}}^0 \sin^2 \theta \, d\theta = -\frac{1}{4} \int_0^{\frac{\pi}{2}} (1 - \cos 2\theta) \, d\theta$$

$$-\frac{1}{4} \left(\theta - \frac{1}{2} \sin 2\theta \right) \Big|_0^{\frac{\pi}{2}}$$

$$\frac{\pi}{4} - \frac{1}{4} \left(\frac{\pi}{2} \right) = \frac{\pi}{8}$$



let $\sin \theta = x$
 $\cos \theta \, d\theta = dx$
 $x=0 \Rightarrow \theta=0$
 $x=1 \Rightarrow \theta=\frac{\pi}{2}$

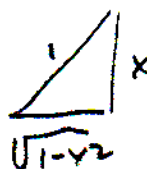
$$III \int \frac{dx}{x^2+4x+8}$$

$$\int \frac{dx}{x^2+4x+4+4}$$

$$\frac{1}{4} \int \frac{dx}{\left(\frac{x+2}{2} \right)^2 + 1}$$

$$2 \cdot \frac{1}{4} \tan^{-1} \left(\frac{x+2}{2} \right) = \frac{1}{2} \tan^{-1} \left(\frac{x+2}{2} \right)$$

$$II_a \int_0^1 \sqrt{1-x^2} \, dx$$



let $x = \sin \theta$
 $dx = \cos \theta \, d\theta$
 $x=0 \Rightarrow \theta=0$
 $x=1 \Rightarrow \theta=\frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}} \cos^2 \theta \, d\theta$$

$$\frac{1}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2\theta) \, d\theta$$

$$\frac{1}{2} \left(\theta + \frac{1}{2} \sin 2\theta \right) \Big|_0^{\frac{\pi}{2}}$$

$$\frac{1}{2} \left(\frac{\pi}{2} + 0 - (0+0) \right) = \frac{\pi}{4}$$

$$\text{(U)} \quad \int_1^2 \frac{x^5 - x^3 + 6x^2 - 6}{x^3 + 2x^2} dx \quad \begin{array}{r} x^2 - 2x + 3 \\ x^3 + 2x^2 \overline{) x^5 - x^3 + 6x^2 - 6} \\ \underline{x^3 + 2x^4} \\ -2x^4 - 4x^3 \\ \underline{-2x^4 - 4x^3} \\ 3x^3 + 6x^2 - 6 \end{array}$$

$$\int_1^2 (x^2 - 2x + 3) dx + \int_1^2 \frac{-6}{x^2(x+2)} dx$$

$$\left. \frac{x^3}{3} - x^2 + 3x \right|_1^2$$

$$\frac{-6}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$$

$$\frac{8}{3} - 4 + 6 - \left(\frac{1}{3} - 1 + 3\right)$$

$$-6 = A(x+2) + B(x+2) + Cx^2$$

$$\frac{8}{3} + 2 - \frac{1}{3} = 2$$

$$x=0 \quad -6 = 2B \quad \therefore B = -3$$

$$x=-2 \quad -6 = 4C \quad \therefore C = -\frac{3}{2}$$

$$x=1 \quad -6 = 3A + 3B + C$$

$$= 3A - 9 - \frac{3}{2}$$

$$\frac{7}{2} = 3A \quad \therefore A = \frac{3}{2}$$

$$\frac{3}{2} \int_1^2 \frac{1}{x} dx - 3 \int_1^2 \frac{1}{x^2} dx - \frac{3}{2} \int_1^2 \frac{dx}{x+2}$$

$$\left. \frac{3}{2} \ln x + \frac{3}{x} - \frac{3}{2} \ln(x+2) \right|_1^2$$

$$\frac{3}{2} \ln 2 + \frac{3}{2} - 3 - \frac{3}{2} (\ln 4 - \ln 3)$$

$$\frac{3}{2} \ln 2 - \frac{3}{2} - \frac{3}{2} \ln \left(\frac{4}{3}\right)$$

$$\frac{3}{2} \ln 2 - \frac{3}{2} - \frac{3}{2} \ln \left(\frac{4}{3}\right)$$

$$\frac{3}{2} (\ln 2 - 1 - \ln \left(\frac{4}{3}\right))$$

$$\frac{3}{2} (\ln \frac{2}{3} - 1)$$

$$\frac{3}{2} (\ln \frac{2}{3} - 1)$$

$$+ \frac{3}{2} (\ln \frac{3}{2} - 1)$$

$$\frac{7}{3} + \frac{3}{2} \ln \frac{3}{2} - \frac{3}{2}$$

$$\frac{7}{3} + \frac{3}{2} (\ln \frac{3}{2} - 1) \quad \frac{7}{3} + \frac{3}{2} \ln \frac{3}{2} - \frac{3}{2}$$

$$1.442 \text{ --- check}$$

$$\frac{7}{3} - \frac{3}{2} \quad \frac{5}{6} + \frac{3}{2} \ln \frac{3}{2}$$

$$\frac{14-9}{6}$$

$$\frac{5}{6} + \frac{3}{2} \ln \frac{3}{2}$$

Extra.

$$\int \frac{x^3}{x^{a-2}} dx$$

If $a=6$

$$\int \frac{x^3}{x^4} dx = \ln|x| + C$$

If $a \neq 6$

$$\int \frac{x^3}{x^{a-2}} dx =$$

$$\int x^{5-a} dx =$$

$$\frac{x^{6-a}}{6-a} + C$$

$$I \int_0^3 \frac{x \, dx}{\sqrt{x+1}} \quad \text{let } x+1=u \quad x=0 \quad u=1$$

$$x=u-1 \quad x=3 \quad u=4$$

$$dx=du$$

$$\int \frac{u-1}{\sqrt{u}} du$$

$$\int \frac{u-1}{\sqrt{u}} du$$

$$\int u^{\frac{1}{2}} - u^{-\frac{1}{2}} du$$

$$\frac{2u^{\frac{3}{2}}}{3} - 2u^{\frac{1}{2}} \Big|_1^4$$

$$\tan \theta = \frac{\sqrt{3}}{1} > \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

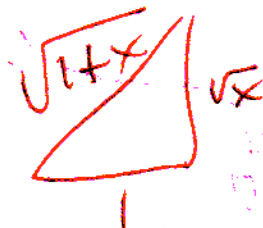
$$\theta = \frac{\pi}{3}$$



Layonna
Hamilton

Kathy Libal
Sec $\theta = \sqrt{x}$

$$\int_0^3 \frac{x \, dx}{\sqrt{1+x}}$$



$$\tan \theta = \sqrt{x}$$

$$\tan^2 \theta = x$$

$$2 \tan \theta \sec^2 \theta \, d\theta = dx$$

$$x=0 \rightarrow \theta=0$$

$$x=3 \rightarrow \theta=\frac{\pi}{3}$$

$$\int_0^{\frac{\pi}{3}} \frac{\tan^2 \theta (2 \tan \theta \sec^2 \theta) \, d\theta}{\sec \theta}$$

$$2 \int \tan^3 \theta \sec \theta \, d\theta$$

$$2 \int (\sec^2 \theta - 1) \tan \theta \sec \theta \, d\theta$$

$$u = \sec \theta$$

$$du = \sec \theta \tan \theta \, d\theta$$

$$2 \int u^2 - 1 \, du$$

$$2 \left[\frac{u^3}{3} - u \right] \Big|_1^2$$

etc.

$$\theta=0 \quad u=1$$

$$\theta=\frac{\pi}{3} \quad u=2$$

93

100	0
90	9
80	4
70	2
60	2
59	4

98

20

1617 8285

20

77.95 actual

Definite integral make
 going easier, because
 in spite of method
~~had~~ know if the "whole
 Process" was 100% successful
 at a glance - It ~~allows~~ allows me
 to look for errors rather
 than to "redo" each and
 every problem for each
 and every student.

2000

100	0
90	5
80	5
70	6
60	0
59	6

99

35

1635

22

74.2

The weak ones have,
 once again, proved
 they are weak.
 Too busy enjoying
 being in Catalans
 to do any calculus
 I guess.

Advanced Placement Calculus

Copy original problem.

Convince *me* that you understand the concept!

Chapter 9 Exam

Name

Ch 9 Calc

Per

Date

3/23/93

also
2000

I

Compute: $\int_0^3 \frac{x \, dx}{\sqrt{x+1}}$

(15 pts)

II

Compute:

a) $\int_0^1 \sqrt{1-x^2} \, dx$

(15 pts)

b) $\int_0^1 x \sin^{-1} x \, dx$

(hint: $-x^2 = (1-x^2) + 1$)

(20 pts)

III

Integrate:

$$\int \frac{dx}{x^2 + 4x + 8}$$

(20 pts)

IV

Compute:

$$\int_1^2 \frac{x^5 - x^3 + 6x^2 - 6}{x^3 + 2x^2} \, dx$$

(30 pts)

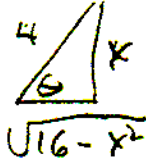
Extra Credit

5 pts

Given a is a constant, compute: $\int \frac{x^3}{x^{a-2}} \, dx$

$$I = \int_2^{2\sqrt{3}} \frac{x^2 dx}{\sqrt{16-x^2}}$$

a) $\text{FNTNT}(x^2/\sqrt{16-x^2}, x, 2, 2\sqrt{3})$ c) let $u = \sqrt{16-x^2}$ $du = \frac{-x dx}{\sqrt{16-x^2}}$
 4.189
 $u^2 = 16-x^2$
 $x^2 = 16-u^2$
 $x = \sqrt{16-u^2}$

b) let $x = 4 \sin \theta$ 
 $dx = 4 \cos \theta d\theta$
 $x=2 \quad \theta = \frac{\pi}{6}$
 $x=2\sqrt{3} \quad \theta = \frac{\pi}{3}$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{16 \sin^2 \theta \cdot 4 \cos \theta d\theta}{4 \cos \theta}$$

$$16 \cdot \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 - \cos 2\theta d\theta$$

$$8 \left(\theta - \frac{1}{2} \sin 2\theta \right) \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$8 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\sqrt{3}}{4} \right)$$

$$8 \cdot \frac{\pi}{6}$$

$$\frac{4\pi}{3} \approx 4.189$$

$$II \int \cos^2 \theta d\theta$$

$$u = \cos \theta \quad du = -\sin \theta d\theta$$

$$du = -\sin \theta d\theta \quad v = \sin \theta$$

$$= \sin \theta \cos \theta + \int \sin^2 \theta d\theta$$

$$+ \int 1 - \cos^2 \theta d\theta$$

$$+ \theta - \int \cos^2 \theta d\theta$$

$$2 \int \cos^2 \theta d\theta = \sin \theta \cos \theta + \theta - \frac{1}{2} \sin 2\theta + \frac{\theta}{2} + C$$

$$x=2 \quad u=2\sqrt{3}$$

$$x=2\sqrt{3} \quad u=2$$

$$\int_2^{2\sqrt{3}} x \frac{x dx}{\sqrt{16-x^2}} \leftarrow \text{Original}$$

$$= \int_{2\sqrt{3}}^2 \sqrt{16-u^2} \cdot du$$

$$+ \int_2^{2\sqrt{3}} \sqrt{16-u^2} du$$

$$u=2 \quad \theta = \frac{\pi}{6} \quad \text{let } u = 4 \sin \theta$$

$$u=2\sqrt{3} \quad \theta = \frac{\pi}{3}$$

$$du = 4 \cos \theta d\theta$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 4 \cos \theta \cdot 4 \cos \theta d\theta$$

$$16 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^2 \theta d\theta$$

$$8 \int 1 + \cos 2\theta d\theta \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$8 \left(\theta + \frac{1}{2} \sin 2\theta \right) \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$8 \cdot \frac{\pi}{3} + \frac{4\sqrt{3}}{2} - 8 \cdot \frac{\pi}{6} - \frac{4\sqrt{3}}{2}$$

$$\frac{4\pi}{3}$$

D. Nothing to worry, all =

$$\text{II} \int \frac{dx}{2+\tan x}$$

c) $\tan x = y$

$$x = \tan^{-1} y$$

$$dx = \frac{1}{1+y^2} dy$$

$$\int \frac{1}{1+y^2} \frac{1}{2+y} dy$$

$$\frac{1}{1+y^2} \cdot \frac{1}{2+y} = \frac{ax+b}{1+y^2} + \frac{c}{2+y}$$

$$1 = (ax+b)(2+y) + c(1+y^2)$$

$$y = -2 \quad 1 = 5c \therefore c = \frac{1}{5}$$

$$y = 0 \quad 1 = 2b + c$$

$$\frac{4}{5} = 2b \therefore b = \frac{2}{5}$$

$$y = 1 \quad 1 = 3(a+b) + 2c$$

$$\frac{5}{5} = 3a + \frac{6}{5} + \frac{2}{5}$$

$$a = -\frac{1}{5}$$

Extra.

$$\int \cot x \csc^2 x dx$$

Take: $u = \cot x \quad dv = \csc^2 x dx$
 $du = -\csc^2 x dx \quad v = -\cot x$

$$= -\cot^2 x - \int \cot x \csc^2 x dx$$

$$2 \int = -\frac{1}{2} \cot^2 x$$

a) $\frac{1}{2+\tan x}$
 $\frac{1}{2\cos x + \sin x}$
 $\frac{\cos x}{2\cos x + \sin x}$

b) we have a mixture of sine and cosine so the $\cos x$ in the numerator cannot be du

$$d) \frac{1}{5} \int \frac{-y+2}{1+y^2} dy + \frac{1}{5} \int \frac{dy}{y+2}$$

$$\Rightarrow \frac{1}{5} \left[-\frac{1}{2} \int \frac{2y}{1+y^2} dy + \int \frac{dy}{1+y^2} + \int \frac{dy}{y+2} \right]$$

$$= \frac{1}{10} \ln|1+y^2| + \frac{2}{5} \tan^{-1} y + \frac{1}{5} \ln|y+2|$$

$$f) \frac{1}{10} \ln|1+\tan^2 x| + \frac{2}{5} x + \frac{1}{5} \ln|\tan x + 2|$$

$$= \frac{1}{10} \ln|\sec x|^2 +$$

$$= \frac{1}{5} \ln|\sec x| + \frac{2}{5} x + \frac{1}{5} \ln|\tan x + 2|$$

$$= \frac{1}{5} \ln \left| \frac{\tan x + 2}{\sec x} \right| + \frac{2}{5} x + C$$

Let $u = \csc x \quad dv = \cot x \csc x dx$
 $du = -\csc x \cot x dx \quad v = -\csc x$

$$= -\csc^2 x - \int \cot x \csc x dx$$

$$2 \int = -\csc^2 x$$

$$\int = -\frac{1}{2} \csc^2 x$$

Since $S^2 + C^2 = 1$

$$1 + \cot^2 x = \csc^2 x$$

$$\frac{1}{2} + \frac{1}{2} \cot^2 x = \frac{1}{2} \csc^2 x$$

so the two answers differ by the constant $\frac{1}{2}$.

$$\int \cos^2 \theta \, d\theta$$

$$u = \cos^2 \theta$$

$$du = -2 \sin \theta \cos \theta \, d\theta$$

$$du = -2 \sin \theta \cos \theta \, d\theta \quad v = \theta$$

$$= \theta \cos^2 \theta + 2 \int \theta \sin \theta \cos \theta \, d\theta$$

$$+ \int \theta \sin 2\theta \, d\theta$$

$$u = \theta$$

$$dv = \sin 2\theta \, d\theta$$

$$du = d\theta$$

$$dv = -\frac{1}{2} \cos 2\theta$$

$$= \frac{1}{2} \theta \cos 2\theta + \int \frac{1}{2} \cos 2\theta \, d\theta$$

$$+ \frac{1}{4} \sin 2\theta$$

$$\theta \cos^2 \theta - \frac{1}{2} \theta \cos 2\theta + \frac{1}{4} \sin 2\theta + C$$

2001

100	4
90	8
80	4
70	4
60	5
59	2

105



30

$$\begin{array}{r} 2182 \\ \hline 27 \end{array}$$

80.8

Without the "30"
the average
was
82.8

3/15/99

100	1
90	3
80	3
70	1
60	1
59	3

101



35

77.2

a great extm! Poor results
I am surprised. Expected
better.

Advanced Placement Calculus

Name

Calc Ch 9

Copy original problem.

Per

Date

2/20/1

Convince *me* that *you* understand the concept!**Calculators may be used on Ia and Id ONLY!**

Chapter 9 Exam

I

Given: $\int_2^{2\sqrt{3}} \frac{x^2 dx}{\sqrt{16-x^2}}$

(40 pts tot)

- Compute the value of the given integral with your calculator.
- In the original, make the substitution $x = 4 \sin \theta$ (including the change in limits) then integrate and evaluate.
- In the original, make the substitution $u = \sqrt{16-x^2}$ then integrate and evaluate.
- Reconcile the three answers above.

IIIntegrate $\int \cos^2 \theta d\theta$ using "Parts"

(20 pts)

IIIGiven: $\int \frac{dx}{2 + \tan x}$. You, no doubt, have noticed that the desired $\sec^2 x$ is not included.

(40 pts tot)

- Convert the fraction, $\frac{1}{2 + \tan x}$ into a single, proper fraction involving sine and cosine.
- Briefly explain why you still do not have the desired $\frac{du}{u}$ situation.
- Let $\tan x = y$. Rewrite the original in terms of y .
- Rewrite your answer to "c" as two integrals with proper numerators.
- Integrate your answer to part "d".
- Show that your answer is equivalent to: $\frac{1}{5} \left(\ln \left| \frac{2 + \tan x}{\sec x} \right| + 2x \right) + c$

Extra Credit ----- 5 pts -----

Given: $\int \cot x \csc^2 x dx$

Jake said, "I use $u = \cot x$." Jed said "I use $u = \csc x$ ". Both integrate their problems (and got different answers, of course.) Resolve the dilemma and explain. Be *very* specific. I'm not interested in generalities.

3/11/02

100	2
20	2
80	1
20	4
60	3
59	12

$$\begin{array}{r} 1401 \\ \hline 22 \end{array}$$

63.7

104

↓
12

Rephrase I + let $z^2 = \frac{1+x}{1-x}$ show that

1996

the given can be rewritten

as $\int \frac{4z^2 dz}{(z^2+1)^2}$

since so much rests on having this correct,

100	1
90	2
80	8
70	2
60	5
59	1

$$\frac{1449}{19} = 76.3$$

102



50

Need to rephrase

Advanced Placement Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

Assume constants of integration to be zero.

Name

Per

Date

Chapter 9 Exam

I

Given: $\int \sqrt{\frac{1+x}{1-x}} dx$

(70 pts tot)

- a) Let $z^2 = \frac{1+x}{1-x}$. Rewrite the *given* in terms of z (hint: solve for x first).
- b) Make a trig substitution on your response to part "a". Integrate. Leave your antiderivative in terms of θ .
- c) List the identities for $\sin^2 \theta$ and $\cos^2 \theta$ which involve $\cos 2\theta$.
Let $x = \cos 2t$. Rewrite dx so that it involves only single angle arguments (ie t rather than $2t$).
Substitute into the original *given*. Integrate. Leave your antiderivative in terms of t .
- d) Multiply the *given* by the fraction: $\frac{\sqrt{1+x}}{\sqrt{1+x}}$. Integrate the result.
- e) Sketch the graph of the *given* (from your grapher) using window $x \in [-2, 2]$ and $y \in [0, 5]$.
- f) Rewrite the *given* with limits of integration $x = -1$ to $x = +1$. What do you notice that might be significant?
Considering the substitution made in part "c", the new limits would be from $z = 0$ to $z = \infty$. In part "b" you made a trig substitution. Compute the new limits of integration in terms of θ and then evaluate your antiderivative.
- g) Assuming the limits of integration of the *given* to be from $x = -1$ to $x = +1$, compute the new limits of integrations and evaluate your antiderivative from part "c".
- h) Assuming the limits of integration of the *given* to be from $x = -1$ to $x = q$, evaluate your antiderivative from part "d". Apply the limit as $q \rightarrow 1^-$.
- i) Use your calculator to compute $\text{FnInt}(\text{the given}, x, -1, 1)$

II

Integrate: $\int \frac{7x^2 - 4x}{(x^2 + 1)(x - 2)} dx$

(15 pts)

III

Use the method "parts" only one time and the fact that $\sin^2 \theta = \sin \theta \cdot \sin \theta$ to integrate: $\int \sin^2 \theta d\theta$

(15 pts)

Extra Credit ----- 5 pts -----

Use the method of partial fractions to rewrite the revised integral for part I a.

$$I \int \sqrt{\frac{1+x}{1-x}} dx =$$

$$1) z^2 = \frac{1+x}{1-x} \quad \int \frac{-4z^2}{(z^2+1)^2} dz$$

$$z^2 - z^2 x = 1+x$$

$$(z^2 - 1) = x(1+z^2)$$

$$x = \frac{z^2 - 1}{z^2 + 1}$$

$$dx = \frac{(z^2+1)2z - (z^2-1)2z}{(z^2+1)^2} dz$$

$$dx = \frac{4z}{(z^2+1)^2} dz$$

$$c) \sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$x = \cos 2t$$

$$dx = -2 \sin 2t dt$$

$$= -4 \sin t \cos t dt$$

$$-4 \int \sqrt{\frac{1+\cos 2t}{1-\cos 2t}} \sin t \cos t dt$$

$$-4 \int \frac{\cos t}{\sin t} \cdot \sin t \cos t dt$$

$$-4 \int \cos^2 t dt$$

$$-2 \int (1 + \cos 2t) dt$$

$$10 \text{PTS} \quad -2 \left(t + \frac{1}{2} \sin 2t \right)$$

$$z^2 = \frac{1+x}{1-x} \quad x \rightarrow -1 \quad z \rightarrow 0$$

$$x \rightarrow 1 \quad z \rightarrow \infty$$

$$z = \tan \theta \quad z \rightarrow 0 \quad \theta \rightarrow 0$$

$$z \rightarrow \infty \quad \theta \rightarrow \frac{\pi}{2}$$

$$b) \quad \sqrt{\frac{z^2+1}{z^2-1}} = z \quad \text{let } z = \tan \theta$$

$$dz = \sec^2 \theta d\theta$$

$$\int \frac{4 \tan^2 \theta}{\sec^4 \theta} \cdot \sec^2 \theta d\theta$$

$$4 \int \sin^2 \theta d\theta$$

$$2 \int (1 - \cos 2\theta) d\theta$$

$$10 \text{PTS} \quad 2\theta - \sin 2\theta$$

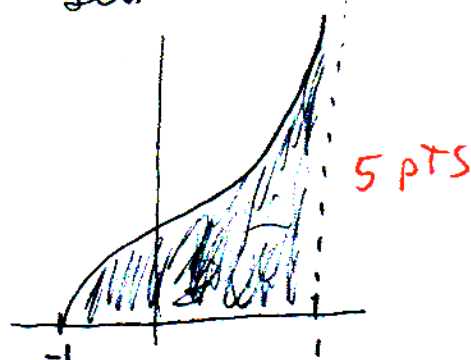
$$d) \int \sqrt{\frac{1+x}{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} dx$$

$$\int \frac{1+x}{\sqrt{1-x^2}} dx$$

$$\int \frac{1}{\sqrt{1-x^2}} dx + \int \frac{x}{\sqrt{1-x^2}} dx$$

$$\sin^{-1} x + \sqrt{1-x^2} \quad 10 \text{PTS}$$

e)



f)

$$\int_{-1}^1 \sqrt{\frac{1+x}{1-x}} dx$$

division by zero at upper bound.

$$2\theta - \sin 2\theta \Big|_0^{\frac{\pi}{2}}$$

$$2 \cdot \frac{\pi}{2} - \sin \pi - (0 - 0)$$

$$\pi$$

10PTS

I G) $x = \cos 2t$
 $x \rightarrow -1 \quad t \rightarrow \frac{\pi}{2}$
 $x \rightarrow 1 \quad t \rightarrow 0$

$-2 \left(\frac{\pi}{2} + \frac{1}{2} \sin 2t \right) \Big|_0^{\frac{\pi}{2}}$
 10pts $0 + 2 \left(\frac{\pi}{2} + \frac{1}{2} \sin \pi \right)$
 π

h) $\sin^{-1} x - \sqrt{1-x^2} \Big|_0^1$

$\sin^{-1} 1 - \sqrt{1-1^2} - (\sin^{-1} 0 - \sqrt{1-0^2})$
 $\lim_{x \rightarrow 1} (\sin^{-1} x - \sqrt{1-x^2} + \frac{\pi}{2})$
 10pts $\sin^{-1} 1 = 0 + \frac{\pi}{2}$
 $\frac{\pi}{2} + \frac{\pi}{2} = \pi$

3.141587126
 3.1416

II $\int \frac{7x^2-4x}{(x^2+1)(x-2)} dx$
 15pts

$\frac{4x+B}{x^2+1} + \frac{C}{x-2}$

$7x^2-4x = (Ax+B)(x-2) + C(x^2+1)$

$x=2 \quad 28-8 = 5C \therefore C=4$
 $x=0 \quad 0 = -2B+C$
 $2B=4 \therefore B=2$
 $x=1 \quad 3 = -(A+B) + 2C$
 $3 = -A-2+8$
 $A=3$

$\left(\frac{3x+2}{x^2+1} + 4 \right) \frac{dx}{x-2}$

$\frac{3}{2} \int \frac{2x dx}{x^2+1} + 2 \int \frac{dx}{x^2+1} + 4 \int \frac{dx}{x-2}$

$\frac{3}{2} \ln(x^2+1) + 2 \tan^{-1} x + 4 \ln|x-2|$

Ex Tra.

$\frac{4z^2}{(z^2+1)^2} = \frac{ax+b}{z^2+1} + \frac{cx+d}{(z^2+1)^2}$

$4z^2 = (ax+b)(z^2+1) + (cx+d)$

$z=0 \quad 0 = b+d$
 $z=1 \quad 4 = 2(a+b) + c+d$
 $z=-1 \quad 4 = 2(-a+b) - c+d$
 $8 = 4b+2d \Rightarrow 4=b+d \therefore b=4, d=-4$

$z=2 \quad 16 = 5(2a+b) + 2c+d$
 $16 = 10a+5b+2c+d$
 $16 = 10a+20+2c-4$
 $5a+c=0$

$0 = 4a+2c$
 $0 = 2a+c$
 $0 = 5a+c$

$0 = 3a \quad a=0, c=0$

$\int \frac{4dz}{z^2+1} - \int \frac{4dz}{(z^2+1)^2}$

III $\int \sin^2 \theta d\theta$
 15pts

$\int \sin \theta \cos \theta d\theta$

$u = \sin \theta \quad du = \cos \theta d\theta$
 $v = \cos \theta \quad dv = -\sin \theta d\theta$

$-\sin \theta \cos \theta + \int \cos^2 \theta d\theta$
 $-\sin \theta \cos \theta + \int (1 - \sin^2 \theta) d\theta$
 $-\sin \theta \cos \theta + \theta - \int \sin^2 \theta d\theta$

$\int \sin^2 \theta = \frac{1}{2} (-\sin \theta \cos \theta + \theta)$

Advanced Placement Calculus

Copy original problem.

Convince me that you understand the concept!

Assume constants of integration to be zero.

Name Calc ch 9
Per _____ Date 3/11/02

Chapter 9 Exam

I Given: $f(x) = \sqrt{\frac{1+x}{1-x}}$. This problem concerns $\int f(x) dx$.

(70 pts tot)

a) Let $z^2 = \frac{1+x}{1-x}$. Show that the $\int f(x) dx$ can be rewritten as $\int \frac{4z^2}{(z^2+1)^2} dz$

b) Make a trig substitution in $\int \frac{4z^2}{(z^2+1)^2} dz$ and then integrate.

Leave your antiderivative in terms of θ .

c) Let $x = \cos 2t$. Rewrite dx so that it involves only single angle arguments (ie t rather than $2t$).

Substitute into $\int \sqrt{\frac{1+x}{1-x}} dx$. Integrate. Leave your antiderivative in terms of t .

d) Multiply $\int \sqrt{\frac{1+x}{1-x}} dx$ by the fraction: $\frac{\sqrt{1+x}}{\sqrt{1+x}}$. Integrate the result.

e) Sketch the graph of $\int \sqrt{\frac{1+x}{1-x}} dx$ (from your grapher) using window $x \in [-2, 2]$ and $y \in [0, 5]$.

f) Rewrite $\int \sqrt{\frac{1+x}{1-x}} dx$ with limits of integration $x = -1$ to $x = +1$. What do you notice that might

be significant? Considering the substitution made in part "a", the new limits would be from $z = 0$ to $z = \infty$. In part "b" you made a trig substitution. Compute the new limits of integration in terms of θ and then evaluate your antiderivative answer to part "b".

g) Consider the changes made in part "c" above including the introduction of limits to be from $x = -1$ to $x = +1$, compute the new limits of integrations in terms of t and then evaluate your antiderivative from part "c".

h) Consider the changes made in part "d" above including the introduction of limits to be from $x = -1$ to $x = q$. Compute the new limits of integration and then evaluate your antiderivative from part "d". Apply the limit as $q \rightarrow 1^-$.

i) Use your calculator to compute $\text{FnInt}(\sqrt{\frac{1+x}{1-x}}, x, -1, 1)$

II Integrate: $\int \frac{7x^2 - 4x}{(x^2 + 1)(x - 2)} dx$ (No calculator) (15 pts)

III Use the method "parts" **only one time** and the fact that $\sin^2 \theta = \sin \theta \cdot \sin \theta$ to integrate: $\int \sin^2 \theta d\theta$ (15 pts)

Extra Credit ----- 5 pts -----

Use the method of partial fractions to rewrite the revised integral for part I a. Do not integrate.

$$I \int \frac{x^2}{\sqrt{1+x}} dx = 2 \int \frac{(u^2-1)^2}{u} du$$

$$a) \text{ let } u = \sqrt{1+x} = 2 \int u^4 - 2u^2 + 1 du$$

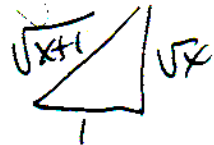
$$= 2 \left(\frac{u^5}{5} - \frac{2u^3}{3} + u \right) = \frac{2}{5} (1+x)^{\frac{5}{2}} - \frac{4}{3} (1+x)^{\frac{3}{2}} + 2(1+x)^{\frac{1}{2}}$$

$$\int \frac{x^2}{\sqrt{1+x}} dx = \int \frac{(u-1)^2}{u} du$$

$$b) \text{ let } u = 1+x = \int \frac{u^2 - 2u + 1}{u} du = \int u - 2 + \frac{1}{u} du = \frac{u^2}{2} - 2u + \ln|u|$$

$$c) \int \frac{x^2}{\sqrt{1+x}} dx$$

$$\text{let } \tan \theta = \sqrt{x}$$



$$\tan^2 \theta = x$$

$$2 \tan \theta \sec^2 \theta d\theta = dx \quad \text{let } u = \sec \theta$$

$$= 2 \int \tan^4 \theta \sec^2 \theta d\theta$$

$$= 2 \int (\sec^2 \theta - 1)^2 \sec^2 \theta d\theta = 2 \int (u^2 - 1)^2 du = 2 \int u^4 - 2u^2 + 1 du = 2 \left(\frac{u^5}{5} - \frac{2u^3}{3} + u \right)$$

$$s^2 + c^2 = 1$$

$$t^2 + 1 = \sec^2$$

1) all 3 are identical

II

$$\int \frac{dx}{x^3+1} = -\frac{1}{3} \int \frac{dx}{x+1} + \frac{1}{3} \int \frac{x-2}{x^2-x+1} dx$$

$$-\frac{1}{3} \ln|x+1| + \frac{1}{6} \int \frac{2x-1}{x^2-x+1} dx$$

$$-\frac{1}{3} \ln|x+1| + \frac{1}{6} \ln|x^2-x+1|$$

$$-\frac{2\sqrt{3}}{3} \tan^{-1} \left[\frac{2}{\sqrt{3}}(x-\frac{1}{2}) \right]$$

$$-\frac{1}{3} \ln|x+1| + \frac{1}{6} \ln|x^2-x+1| - \frac{1}{\sqrt{3}} \tan^{-1} \left[\frac{2}{\sqrt{3}}(x-\frac{1}{2}) \right]$$

$$= \frac{2}{5} (\sec^5 \theta) - \frac{4}{3} \sec^3 \theta + 2 \sec \theta = \frac{2}{5} (x+1)^{\frac{5}{2}} - \frac{4}{3} (x+1)^{\frac{3}{2}} + 2(x+1)^{\frac{1}{2}}$$

$$\frac{2}{3} \frac{4}{6}$$

$$\frac{1}{6} \frac{1}{6}$$

$$-\frac{1}{\sqrt{3}} \int \frac{dx}{\frac{4}{3} \left(\frac{4}{3} (x-\frac{1}{2})^2 + 1 \right)}$$

$$-\frac{2}{3} \int \frac{dx}{\left(\frac{2}{\sqrt{3}} (x-\frac{1}{2})^2 + 1 \right)}$$

Don't put any
-105
-5
got this

$$\frac{dx}{x^2-x+1} = \frac{dx}{(x-\frac{1}{2})^2 + \frac{3}{4}} = \frac{\frac{4}{3} dx}{\frac{4}{3} (x-\frac{1}{2})^2 + 1}$$

III) 4) $\int \frac{x^2}{(7x-3)^2} dx$

$$49x^2 - 42x + 9 \mid x^2$$

$$x^2 - \frac{42}{49}x + \frac{9}{49}$$

not a plane $\frac{1}{49}$
-5-

$$= \frac{1}{49} \int dx + \frac{1}{49} \int \frac{42x-9}{(7x-3)^2} dx$$

$$\frac{42x-9}{(7x-3)^2} = \frac{a}{7x-3} + \frac{b}{(7x-3)^2}$$

$$= \frac{1}{49} \int dx + \frac{6}{49} \int \frac{dx}{7x-3} + \frac{9}{49} \int \frac{dx}{(7x-3)^2}$$

$$42x-9 = a(7x-3) + b$$

$$9 = b$$

$$\text{if } x=0 \quad -9 = -3a + 9 \quad a = 6$$

$$= \frac{1}{49} x + \frac{6}{7^3} \ln|7x-3| - \frac{9}{7^3(7x-3)}$$

$$\frac{1}{7^3} \left[7x + 6 \ln|7x-3| - \frac{9}{7x-3} \right]$$

Extra

$$\int_0^1 \tau h(\tau) d\tau =$$

$$u = \tau \quad dv = h(\tau) d\tau$$

$$du = d\tau \quad v = \int h(\tau) d\tau = g(\tau)$$

$$\tau g(\tau) \Big|_0^1 - \int_0^1 g(\tau) d\tau$$

$$\tau g(\tau) \Big|_0^1 - f(\tau) \Big|_0^1$$

$$1(0-0) - (1-0) = -1$$

B) $\int \frac{x^2}{(7x-3)^2} dx$ let $u = 7x-3$
 $\frac{1}{7}(u+3) = x$
 $\frac{1}{7} du = dx$
 $\frac{1}{7^2}(u+3)^2 = x^2$

$$\frac{1}{7^3} \int \frac{(u+3)^2}{u^2} dx$$

$$\frac{1}{7^3} \int \frac{u^2 + 6u + 9}{u^2} dx$$

$$\frac{1}{7^3} \int \left(1 + \frac{6}{u} + \frac{9}{u^2} \right) dx$$

$$\frac{1}{7^3} \left(u + 6 \ln|u| - \frac{9}{u} \right)$$

$$\frac{1}{7^3} \left(7x-3 + 6 \ln|7x-3| - \frac{9}{7x-3} \right)$$

c) B differs by the constant $-\frac{3}{343}$.

didn't get either A nor B
But reasonably close -15

3/17/97

100	0	98
90	6	80.7
80	4	
70	6	
60	2	
59	1	45

maybe I've carried too
many weak students too
long

3/17/03

100	0	0	0	98
90	2	3		20
80	5	1		
70	4	9		
60	3	1		
59	1	3		
1125				1217
15				17

73.2

75 71.6

~~They give answers to~~ They
They can't memorize
must learn!

Advanced Placement Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

Assume constants of integration to be zero.

No calculators.

Name

Calc Ch 9

Per

Date

2003

Chapter 9 Exam

I

Given $\int \frac{x^2}{\sqrt{1+x}} dx$

(35 pts tot)

- Integrate the given integral using the substitution: $u = \sqrt{1+x}$
- Integrate the given integral using the substitution: $u = 1+x$
- Integrate the given integral using the substitution: $\tan \theta = \sqrt{x}$
- Reconcile any differences in the answers in the problems above.

II

When using the method of partial fractions on $\int \frac{dx}{x^3+1}$

becomes: $-\frac{1}{3} \int \frac{dx}{x+1} + \frac{1}{3} \int \frac{x-2}{x^2-x+1} dx$. Finish the integration.

(30 pts)

III

Given $\int \frac{x^2}{(7x-3)^2} dx$

(35 pts tot)

- Integrate using the method partial fractions.
- Integrate without using the method partial fractions.
- Reconcile any differences in the above two answers.

change to

$g'(t)$
 $\uparrow$
 t

Extra Credit ----- 5 pts -----

Given: $f'(t) = g(t)$, $h(t) = g'(x)$, $t \in [0,1]$ and
 $f(0) = 0$, $f(1) = 1$, $g(0) = 0$, $g(1) = 0$, $h(0) = 0$, $h(1) = 0$

Find $\int_0^1 t h(t) dt$

AP Calculus AB CHAPTER 9 2003

This is some new stuff!

	Exam Ch 7 & 8 # 1 R/C Pg 307 - 312 # 2 Pg 312 1 - 27 odd, 28 A	# 3 Pg 312 2 - 26 even 30, 31, 33, 34, 36 # 4 R/C Pg 313 - 318	# 5 Pg 318 - 319 1 - 37 odd, 41 - 45 odd
# 6 Pg 318-319 2 - 44 even, 46 - 51	# 7 Pg 322 1 - 9, 32, 33 Indicate the formula number you used for each.	# 8 Pg 325 - 326 1 - 11, 13, 14, 23, 24 Pblm 6: Change lower bound from zero to 1. Substitute & Evaluate Check using your grapher	R/C Pg 332 - 336 # 10 Pg 336 1, 3, 5 - 8, 11, 16, 17 ALSO determine true or false the two equations in the lower right portion of this HW sked. # 11
Important lecture # 12 Re-Write today's class notes, legibly. Include "comments" which explain the significance of each of the steps. ALSO # 13 Pg 342 16 - 30A	# 14 Graph $F(x) = \sin^2 x$ and $G(x) = \cos^2 x$ on the axis provided on back of sked.	Important lecture # 15 Pg 350 1 - 13, 16, 19 # 16 Pg 354 - 355 1 - 5, 10, 13, 16 Do problem 13 first using $3x = 2 \tan \theta$ then re-do using $u = 4 + 9x^2$	GQ Pg 356 - 357 1 - 8, 10, 11, 13, 14 # 17 Start working on the review section listed on Monday.
# 18, 19, 20, 21, 22 Review Section:. Entire Review section will be done as a unit, complete, in order and correct and turned-in on the day of the exam. Do Your Own Work ! Get help AFTER you have made <i>several</i> attempts. Pages 357 - 360 : 1 - 12A, 13B, 14A, 17 - 20, 27 - 29, 34 - 37, 40 - 43, 44, 46, 47, 49, 53, 148, 151 - 154, 159	Hint on pblm 11 ... let $u = \cos \theta$	Put last year's exam on the chalkboards.	
3 / 10	3 / 11	3 / 12	3 / 13
EXAM Chapter 9 2003 3 / 17		1) $\frac{3}{x-1} + \frac{1}{x-2} = \frac{?}{x^2-3x+2}$ 2) $\frac{1}{x-2} - \frac{3}{x^2-4x+4} + \frac{2}{2x-1} = \frac{?}{2x^3-9x^2+12x-4}$	3 / 14

REVIEW SECTION ANSWERS AND/OR HINTS

1				
$\frac{3}{4}$				
$\frac{1}{2}$				
$\frac{1}{4}$				

$$f(x) = \sin^2 x \quad \left(\frac{\pi}{8} \text{ (pencil)}\right) \quad g(x) = \cos^2 x \quad \left(\frac{\pi}{4} \text{ (pen)}\right) \quad \frac{3\pi}{8} \quad \frac{\pi}{2}$$

Rewrite each function in terms of sine or cosine but without any exponents. Give a detailed explanation of how to graph each function *without* the aid of any kind of calculator.

2) $x \ln x - x$; $\frac{1}{2} (\ln x)^2$

4) let $u = e^x$; $3 \sec^2 \theta = x^2$

6) $\frac{5}{2} x^2 + 3 \ln|x-1| + \ln(x^2 + 1) - \tan^{-1} x$

12a) $-\cos \theta + \frac{2}{3} \cos^3 \theta - \frac{1}{5} \cos^5 \theta$

$\tan \theta - \theta - \frac{1}{3} \tan^3 \theta + \frac{1}{5} \tan^5 \theta$

18) $x + \frac{1}{x-1} + \frac{x-1}{x^2 + x + 1}$

20) $1 + \frac{1}{x-1} + \frac{2}{x+1} - \frac{1}{x^2 + 1}$

25) Briefly explain ALL possibilities

29) Page 351 case 3

30) Page 349 eg 8

34) $\frac{4}{3} \sqrt{\sqrt{x} + 2} (\sqrt{x} + 4)$

43) $\sin 3x \cos 3x = \frac{1}{2} \sin 6x$

46) $\frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| + \frac{1}{2} \tan^{-1} x$

150) So, who needs tables, anyway?

$$I \int_2^3 \frac{2+2u-u^2}{(u-1)(u^2+u+1)} du = \int \frac{A du}{u-1} + \int \frac{Bx+C}{u^2+u+1} du$$

$$2+2u-u^2 = A(u^2+u+1) + (Bx+C)(u-1)$$

$$\begin{aligned} u=1 & \quad 3=3A \quad \therefore A=1 \\ u=0 & \quad 2=1-C \quad \therefore C=-1 \\ u=-1 & \quad -1=1+(C-B)(-2) \quad 1=C-B \quad B=-2 \end{aligned}$$

$$\int_2^3 \frac{du}{u-1} + \int_2^3 \frac{-2u-1}{u^2+u+1} du \quad 15 \text{pts}$$

$$\ln(u-1) \Big|_2^3 - \ln(u^2+u+1) \Big|_2^3 = \ln 2 - \ln 1 - [\ln 13 - \ln 7] = \ln \frac{14}{13}$$

$$II \int_0^1 \frac{x^2 \ln x}{\sqrt{4-x^2}} dx \quad \begin{array}{c} \text{triangle} \\ \sqrt{4-x^2} \end{array} \quad \begin{array}{l} x=2\sin\theta \\ dx=2\cos\theta d\theta \\ \sqrt{4-x^2}=2\cos\theta \end{array} \quad \frac{4}{2} \int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{\cos\theta} \cdot 2\cos\theta d\theta = \quad 15 \text{pts}$$

$$4 \frac{1}{2} \int_0^{\frac{\pi}{6}} 1 - \cos 2\theta d\theta = 2 \left[\theta - \frac{1}{2} \sin 2\theta \right] \Big|_0^{\frac{\pi}{6}} = 2 \left[\frac{\pi}{6} - \frac{1}{2} \frac{\sqrt{3}}{2} \right] = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$$

$$III \int_0^{\pi} x \sin x dx \quad \begin{array}{l} u=x \quad v=-\cos x \\ du=dx \quad dv=\sin x dx \end{array}$$

$$-x \cos x \Big|_0^{\pi} + \int_0^{\pi} \cos x dx = -x \cos x + \sin x \Big|_0^{\pi} = -(-\pi) + 0 - 0 = \pi \quad 15 \text{pts}$$

$$IV \int_0^3 \frac{du}{9+u^2} = \frac{1}{9} \int_0^3 \frac{du}{1+\frac{u^2}{9}} = \frac{1}{9} \int_0^3 \frac{du}{1+(\frac{u}{3})^2} \Rightarrow \frac{1}{9} \cdot 3 \tan^{-1}\left(\frac{u}{3}\right) \Big|_0^3$$

$$\frac{1}{3} \left[\tan^{-1} 1 - \tan^{-1} 0 \right] = \frac{1}{3} \left[\frac{\pi}{4} - 0 \right] = \frac{\pi}{12} \quad 15 \text{pts}$$

$$\underline{V} \int_0^{\frac{\pi}{3}} \frac{\tan \theta}{\sec \theta} d\theta = \int_0^{\frac{\pi}{3}} \frac{\sin \theta}{\cos \theta} d\theta = -\cos \theta \Big|_0^{\frac{\pi}{3}} = -\frac{1}{2} + 1 = \frac{1}{2} \quad 10 \text{PTS}$$

$$\underline{VI} \int_0^1 2^x dx = \int_0^1 e^{x \ln 2} dx = \frac{1}{\ln 2} e^{x \ln 2} \Big|_0^1 = \frac{1}{\ln 2} 2^x \Big|_0^1$$

$$\frac{1}{\ln 2} [2 - 1] = \frac{1}{\ln 2} \quad 10 \text{PTS}$$

$$\underline{VII} \int_0^{\frac{\pi}{2}} \sin^5 \theta \cos^3 \theta d\theta = \int_0^{\frac{\pi}{2}} \sin^4 \theta \cos^2 \theta \cos \theta d\theta = \int_0^{\frac{\pi}{2}} \sin^4 \theta (1 - \sin^2 \theta) \cos \theta d\theta$$

$$u = \sin \theta$$

$$du = \cos \theta d\theta$$

$$\theta = 0 \quad u = 0$$

$$\theta = \frac{\pi}{2} \quad u = 1$$

$$\int_0^1 u^4 (1 - u^2) du = \int_0^1 u^4 - u^6 du =$$

$$\left[\frac{u^5}{5} - \frac{u^7}{7} \right]_0^1 = \frac{1}{5} - \frac{1}{7} = \frac{2}{35} \quad 10 \text{PTS}$$

$$\underline{VIII} \int_0^{\frac{\pi}{2}} \frac{\sqrt{1 + \cos \theta}}{\sqrt{1 - \cos \theta}} d\theta$$

$\sin \theta > 0$
 $\ln 187$
 $\text{Quad.}, \text{Not}$

Extra
 $\int_1^2 \frac{x+1}{x^2+2x} dx \quad \frac{A}{x} + \frac{B}{x+2}$

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{1 - \cos^2 \theta}}{\sqrt{1 - \cos \theta}} d\theta = \int_0^{\frac{\pi}{2}} \frac{\sin \theta d\theta}{\sqrt{1 - \cos \theta}}$$

$$u = 1 - \cos \theta \quad \theta = 0 \quad u = 0$$

$$du = \sin \theta d\theta \quad \theta = \frac{\pi}{2} \quad u = 1$$

$$\int_0^1 u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} \Big|_0^1 = 2 \quad 10 \text{PTS}$$

$$x+1 = (x+2)A + Bx$$

$$x=0 \quad 1 = 2A \quad \therefore A = \frac{1}{2}$$

$$x=-2 \quad -1 = -2B \quad \therefore B = \frac{1}{2}$$

$$\frac{1}{2} \int_1^2 \frac{dx}{x} + \frac{1}{2} \int_1^2 \frac{dx}{x+2} =$$

$$\frac{1}{2} (\ln|x| + \ln|x+2|) \Big|_1^2$$

$$\frac{1}{2} (\ln 2 + \ln 4 - \ln 1 - \ln 3)$$

$$\frac{1}{2} \ln \frac{8}{3}$$

$$\ln 2\sqrt{\frac{2}{3}}$$

89

100	5	104
90	8	}
80	5	
70	6	
60	0	
59	2	
		29

87.6

without the 29, ave = 90.3

3/29/85

104
103-2
102
101
100-2

7

95
94
91-2

4

88
86
84

3

77
76-2
72
71

5

65
64
63
62-2
60

6

57

1

$\frac{2047}{26}$

78.7

A.P. CALCULUS A.B

NAME Calculus Ch 9Convince ME that *YOU* know the concept!PER _____ DATE 3/20/89

Chapter 9

Compute:

$$\text{I} \quad \int_2^3 \frac{2+2u-u^2}{(u-1)(u^2+u+1)} du$$

15 pts

$$\text{II} \quad \int_0^1 \frac{x^2}{\sqrt{4-x^2}} dx$$

15 pts

$$\text{III} \quad \int_0^\pi x \sin x \, dx$$

15 pts

$$\text{IV} \quad \int_0^3 \frac{du}{9+u^2}$$

15 pts

$$\text{V} \quad \int_0^{\frac{\pi}{3}} \frac{\tan \theta \, d\theta}{\sec \theta}$$

10 pts

$$\text{VI} \quad \int_0^1 2^x \, dx$$

10 pts

$$\text{VII} \quad \int_0^{\frac{\pi}{2}} \sin^5 \theta \cos^3 \theta \, d\theta$$

10 pts

$$\text{VIII} \quad \int_0^{\frac{\pi}{2}} \sqrt{1+\cos \theta} \, d\theta$$

10 pts

EXTRA CREDIT ----- 5 points

COMPUTE:

$$\int_1^2 \frac{1+x}{x^2+2x} dx$$

Write final answer in
most compact form.

I $\int \sec \theta d\theta = \ln |\sec \theta + \tan \theta|$ total 30

A) $\int \frac{d\theta}{\cos \theta} = \int \frac{\cos \theta d\theta}{\cos^2 \theta} = \int \frac{\cos \theta d\theta}{1 - \sin^2 \theta}$ $u = \sin \theta$
 $du = \cos \theta d\theta$

$\int \frac{du}{1-u^2} \Rightarrow \frac{1}{1-u^2} = \frac{A}{1-u} + \frac{B}{1+u}$ $1 = A(1+u) + B(1-u)$
 $u=1 \quad 1 = 2A \therefore A = \frac{1}{2}$
 $u=-1 \quad 1 = 2B \therefore B = \frac{1}{2}$

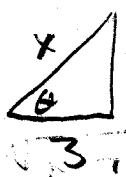
$\frac{1}{2} \int \frac{du}{1+u} + \frac{1}{2} \int \frac{du}{1-u} = \frac{1}{2} \ln |1+u| - \frac{1}{2} \ln |1-u|$

$\frac{1}{2} \ln \left| \frac{1+u}{1-u} \right| \Rightarrow \frac{1}{2} \ln \left| \frac{1+\sin \theta}{1-\sin \theta} \right| \frac{1+\sin \theta}{1+\sin \theta} \Rightarrow \frac{1}{2} \ln \frac{(1+\sin \theta)^2}{\cos^2 \theta}$

$\ln \left| \frac{1+\sin \theta}{\cos \theta} \right| = \ln |\sec \theta + \tan \theta|$

B)

$\int \frac{dx}{\sqrt{x^2-9}}$



$\sec \theta = \frac{x}{3}$

$\tan \theta = \frac{\sqrt{x^2-9}}{3}$

$3 \sec \theta = x$

$3 \tan \theta = \sqrt{x^2-9}$

$3 \sec \theta \tan \theta = dx$

$\frac{3}{3} \int \frac{\sec \theta \tan \theta d\theta}{\tan \theta} = \int \sec \theta = \ln |\sec \theta + \tan \theta|$
 $= \ln \left| \frac{x}{3} + \frac{\sqrt{x^2-9}}{3} \right|$

II

A) $\int \frac{2 \sin \theta d\theta}{\cos^3 \theta}$ total 20

$u = \cos \theta$
 $du = -\sin \theta d\theta$

$-2 \int \frac{du}{u^3} = -\frac{2}{-2} \frac{1}{u^2} = \frac{1}{\cos^2 \theta} = \sec^2 \theta$

B) $\int \frac{2 \sin \theta d\theta}{\cos^3 \theta} = 2 \int \tan \theta \sec^2 \theta d\theta$
 $u = \sec \theta$
 $du = \sec \theta \tan \theta d\theta$
 $= 2 \int u du = u^2 = \sec^2 \theta$

C) Since $S^2 + C^2 = 1$ thus both answers differ by only a constant (the 1)
 The constant is zero.
 The derivatives of the constant are valid.

III 4) $\int \frac{3x^2+x+1}{(x+1)(x^2+2)} dx$ $3x^2+x+1 = A(x^2+2) + (Bx+C)(x+1)$
 15 $x=-1$ $3 = 3A \therefore A=1$
 $x=0$ $1 = 2A + C \therefore C = -1$

$\int \frac{dx}{x+1} + \int \frac{2x-1}{x^2+2}$ $x=1$ $5 = 3A + 2(B+C)$
 $5 = 3 + 2B - 2$
 $4 = 2B \therefore B=2$

$\ln|x+1| + \int \frac{2x dx}{x^2+2} - \int \frac{dx}{x^2+2}$

$\ln|x+1| + \ln(x^2+2) - \frac{1}{2} \int \frac{dx}{(\frac{x}{\sqrt{2}})^2 + 1}$

$\ln|x+1| + \ln(x^2+2) - \frac{\sqrt{2}}{2} \tan^{-1}(\frac{x}{\sqrt{2}})$

IV $\int x^2 \sqrt{3-x} dx$

$u = 3-x$

$du = -dx$

$x = 3-u$

$x^2 = 9 - 6u + u^2$

20

$-\int (9 - 6u + u^2) u^{\frac{1}{2}} du = \int -9u^{\frac{1}{2}} + 6u^{\frac{3}{2}} - u^{\frac{5}{2}} du$

$= -9 \frac{2}{3} u^{\frac{3}{2}} + 6 \frac{2}{5} u^{\frac{5}{2}} - \frac{2}{7} u^{\frac{7}{2}} - 2 \text{ still in } u$

$= -6u^{\frac{3}{2}} + \frac{12}{5} u^{\frac{5}{2}} - \frac{2}{7} u^{\frac{7}{2}} =$

$= -6(3-x)^{\frac{3}{2}} + \frac{12}{5} (3-x)^{\frac{5}{2}} - \frac{2}{7} (3-x)^{\frac{7}{2}}$

Extra $f(x) = (|x|-x) \sqrt[3]{9x}$ Since $|x| = x^2$

5 $= ((x^2)^{\frac{1}{2}} - x) (9x)^{\frac{1}{3}}$

$f'(x) = (9x)^{\frac{1}{3}} \left[\frac{1}{2} (x^2)^{\frac{1}{2}} (2x) - 1 \right] + [(x^2)^{\frac{1}{2}} - x] \frac{1}{3} (9x)^{\frac{1}{3}} \cdot 9$

$= (9x)^{\frac{1}{3}} \left(\frac{x}{|x|} - 1 \right) + \frac{3(|x|-x)}{(9x)^{\frac{2}{3}}}$

$f'(-3) = (-2)^{\frac{1}{3}} \left(\frac{-3}{3} - 1 \right) + \frac{3(1-3)}{(-2)^{\frac{2}{3}}}$

$= -3(-2) + \frac{3(6)}{9} = 6 + 2 = 8$

OPPI P.B. 262

1001	4	104
901	2	
80	1	
70	5	
60	0	
59	1	

↓
55

84.5

Excellent
Exam

ADVANCED PLACEMENT CALCULUS AB
Convince ME that YOU know the concept!
Copy original problem. SHOW ALL WORK!

NAME Calc ch 9 1990
PER _____ DATE _____

CHAPTER 9 EXAM

Assume constants of integration to be zero.

I A) PROVE:
(Use method of "partial fractions") $\int \sec \theta \, d\theta = \ln | \sec \theta + \tan \theta |$ (30 pts tot)

B) Using "A", integrate: $\int \frac{dx}{\sqrt{x^2 - 9}}$

II Given: $\int \frac{2 \sin \theta \, d\theta}{\cos^3 \theta}$ (20 pts tot)

A) Integrate using $u = \cos \theta$

B) Integrate using $u = \tan \theta$

C) Completely resolve ANY discrepancies between parts "A" and "B".

III Compute:

A) $\int \frac{3x^2 + x + 1}{(x+1)(x^2+2)} \, dx$

B) $\int 2^x \, dx$

IV Do NOT use "parts". Integrate: $\int x^2 \sqrt{3-x} \, dx$ (20 pts)

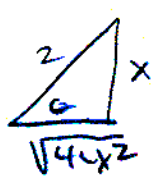
EXTRA CREDIT..... 5 pts

Given $f(x) = (|x| - x) \sqrt[3]{9x}$

Find $f'(-3)$

①

$$10 \int \frac{dx}{(4-x^2)^{3/2}} = \int \frac{dx}{(4-x^2)\sqrt{4-x^2}} = \int \frac{2 \sin \theta d\theta}{8 \cos^3 \theta} = \frac{1}{4} \int \sec^2 \theta d\theta =$$



$$\text{let } x = 2 \sin \theta$$

$$\sqrt{4-x^2} = 2 \cos \theta$$

$$dx = 2 \cos \theta d\theta$$

$$\frac{1}{4} \tan \theta \Rightarrow \frac{x}{4} \frac{\sqrt{4-x^2}}{(4-x^2)}$$

②

$$15 \int \frac{dx}{\sqrt{2+\sqrt{x}}} = \int \frac{2(u-2)du}{u^{1/2}} = 2 \int u^{1/2} du - 4 \int u^{-1/2} du$$

$$\text{let } u = 2 + \sqrt{x}$$

$$(u-2)^2 = x$$

$$2(u-2)du = dx$$

$$\frac{2 \cdot 2}{3} u^{3/2} - 4 \cdot 2 u^{1/2} = \frac{4}{3} (2+\sqrt{x})^{3/2} - 8 \sqrt{2+\sqrt{x}}$$

③

$$15 \int \frac{\sqrt[5]{1+x}}{\sqrt[3]{1+x}} dx = \int \frac{u^3 \cdot 15 u^{14} du}{u^5} = 15 \int u^{12} du = \frac{15}{13} u^{13}$$

$$\text{let } u^{15} = 1+x \quad u = (1+x)^{1/15}$$

$$15 u^{14} du = dx$$

$$\frac{15}{13} (1+x)^{13/15}$$

④

$$20 \int \frac{5x^4 - 5x^3 + 10x^2 - 8x + 4}{(x^2+1)(x-1)} = \int 5x dx + \int \frac{5x^2 - 3x + 4}{(x^2+1)(x-1)}$$

$$\text{let } 5x^2 - 3x + 4 = (x-1)(ax+b) + (x^2+1)(c)$$

$$\parallel 6 = 2c \therefore c = 3$$

$$\parallel 4 = -b + 3 \quad b = -1$$

$$\parallel 12 = -2(-a+b) + 6$$

$$6 = -2(-a+b)$$

$$-3 = -a + b$$

$$-3 = -a - 1 \quad a = 2$$

$$\begin{array}{r} 5x^4 - 5x^3 + 10x^2 - 8x + 4 \\ \underline{5x^4 - 5x^3 + 5x^2 - 5x} \\ 5x^2 - 3x + 4 \end{array}$$

$$\frac{5}{2}x^2 + \int \frac{2x-1}{x^2+1} dx + \int \frac{3}{x-1} dx$$

$$\frac{5}{2}x^2 + \int \frac{2x}{x^2+1} dx - \int \frac{1}{x^2+1} dx + \int \frac{3}{x-1} dx$$

$$\frac{5}{2}x^2 + \ln(x^2+1) + \tan^{-1}x + 3 \ln|x-1|$$

$$\textcircled{5} \int x \tan^{-1} x \, dx = \frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{x^2+1} \, dx$$

W
let $u = \tan^{-1} x$ $v = \frac{1}{2} x^2$
 $du = \frac{1}{x^2+1} dx$ $dv = x \, dx$

$$\frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2} \int dx + \frac{1}{2} \int \frac{dx}{x^2+1}$$

$$x^2+1 \overline{\begin{array}{r} 1 \\ x^2+1 \end{array}}$$

$$\frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2} x + \frac{1}{2} \tan^{-1} x$$

$$\textcircled{6} \int \frac{e^{3x} + 6e^{2x} + 9e^x \, dx}{(e^x - 1)^3} = \int \frac{u^2 + 6u + 9 \, du}{(u-1)^3} \quad \frac{A}{u-1} + \frac{B}{(u-1)^2} + \frac{C}{(u-1)^3}$$

let $u = e^x$ let $u^2 + 6u + 9 = A(u-1)^3 + B(u-1) + C$

$$du = e^x \, dx$$

$$1) \quad 16 = C$$

$$0) \quad 9 = A - B + 16$$

$$-7 = A - B$$

$$-1) \quad 4 = 4A - 2B + 16$$

$$\begin{array}{r} -12 \\ -6 = 24 - B \end{array}$$

$$\begin{array}{r} B - A = 7 \\ -13 + 2A = -6 \\ \hline A = 1 \\ B = 8 \end{array}$$

$$\int \frac{du}{u-1} + \int \frac{8 \, du}{(u-1)^2} + \int \frac{16 \, du}{(u-1)^3} = \ln|u-1| - \frac{8}{u-1} - \frac{8}{(u-1)^2}$$

$$\ln|e^x - 1| - \frac{8}{e^x - 1} - \frac{8}{(e^x - 1)^2}$$

$$\textcircled{7} \int_0^1 \sqrt{1+\cos \theta}^3 \sin \theta \, d\theta = - \int_2^1 u^{3/2} \, du = \int_1^2 u^{3/2} \, du = \left. \frac{2}{5} u^{5/2} \right|_1^2$$

let $u = 1 + \cos \theta$
 $du = -\sin \theta \, d\theta$

$$\frac{2 \cdot 2^{5/2}}{5} - \frac{2}{5} = \frac{2}{5} (4\sqrt{2} - 1)$$

Extra
 $\int_0^2 \left(\int_{x^3}^{x^2} xy \, dy \right) dx = \int_0^2 \left. \frac{xy^2}{2} \right|_{x^3}^{x^2} dx = \int_0^2 \left(2x^5 - \frac{x^7}{2} \right) dx =$

$$\left. \frac{x^6}{3} - \frac{x^8}{16} \right|_0^2 = \frac{2^6}{3} - \frac{2^8}{16} = \frac{2^6}{3} - 2^4 =$$

$$2^4 \left(\frac{4}{3} - 1 \right) = \frac{16}{3}$$

91

100	0
90	6
80	7
70	8
60	2
59	2

$$\frac{2008}{25} = 80.32$$

Excellent exam

only error was dx fen
do on # 7.the extra were probably
too hard.

83

$$\frac{1692}{25} = 67.68$$

Students chose to
Play instead of work -
a rude awakening -
all plans will not be
ones ~~you~~ They've already
seen -

Excellent exam -
no changes.

100	0	92
90	1	
80	9	
70	3	
60	5	
59	7	
		22

Copy the original problem.

Convince me that you understand the concept.

Per

Date

83

CHAPTER #9

Assume that constants of integration are zero.

Calculate:

1) $\int \frac{dx}{(4-x^2)^{\frac{3}{2}}}$

(10 pts)

2) $\int \frac{dx}{\sqrt{2+\sqrt{x}}}$

(15 pts)

3) $\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1+x}} dx$

(15 pts)

4) $\int \frac{5x^4 - 5x^3 + 10x^2 - 8x + 4}{(x^2 + 1)(x-1)} dx$

(20 pts)

5) $\int x \tan^{-1} x dx$

(10 pts)

6) $\int \frac{e^{3x} + 6e^{2x} + 9e^x}{(e^x - 1)^3} dx$

(15 pts)

7) $\int_0^{\frac{\pi}{2}} \sin \theta \sqrt{(1 + \cos \theta)^3} d\theta$

(15 pts)

EXTRA CREDIT

5 pts

Evaluate:

$$\int_0^2 \left(\int_{x^3}^{2x^2} xy dy \right) dx$$

104 - 2

95

90 - 2 are 90.7

89

88

87

80 - 2

Good easy extm.
Student's worked hard
since I called Hw.
This "New" Hw procedure
seems excellent. I may
use it in Trig (Analysis)
Next year.

$[1, 0] \times 3$ $(1/3) = 0.333$, $(0/3) = 0.000$ $(1/3) = 0.333$
 $(0/3) = 0.000$, $(1/3) = 0.333$, $(0/3) = 0.000$ $(1/3) = 0.333$
 $(0/3) = 0.000$, $(0/3) = 0.000$

$\frac{1}{3} = 0.333$

26.7

104

102

101

100 - 2

99

96

95

93

88 - 2

75

62

57

50

40

$$\frac{1350}{16} = 84.4$$

Certainly the
bottom 3 are in
the wrong class....
as in 88, still a very easy
exam.

$$I \quad 4) \int \frac{x^2 dx}{\sqrt{1+x}} = 2 \int \frac{(u^4 + 2u^2 + 1)u du}{u}$$

$$u = \sqrt{1+x} \quad = 2 \left[\frac{u^5}{5} + \frac{2u^3}{3} + u \right]$$

$$u^2 = 1+x$$

$$u^2 + 1 = x$$

$$(u^2 + 1)^2 = x^2$$

$$2u du = dx$$

$$= 2 \left[\frac{1}{5}(1+x)^{\frac{5}{2}} + \frac{2}{3}(1+x)^{\frac{3}{2}} + (1+x)^{\frac{1}{2}} \right]$$

$$B) \quad = \int \frac{u^2 - 2u + 1}{u^{\frac{1}{2}}} du = \int u^{\frac{3}{2}} - 2u^{\frac{1}{2}} + u^{-\frac{1}{2}} du =$$

$$u = 1+x \quad \frac{2}{5} u^{\frac{5}{2}} - \frac{4}{3} u^{\frac{3}{2}} + 2u^{\frac{1}{2}} = \frac{2}{5}(1+x)^{\frac{5}{2}} - \frac{4}{3}(1+x)^{\frac{3}{2}} + 2(1+x)^{\frac{1}{2}}$$

$$du = dx \quad x^2 = u^2 - 2u + 1$$

$$c) \quad x = \tan^2 \theta \quad = 2 \int \frac{\tan^5 \theta \sec^2 \theta d\theta}{\sec \theta} = 2 \int \tan^4 \theta \sec \theta d\theta =$$

$$dx = 2 \tan \theta \sec^2 \theta$$

$$s^2 + c^2 = 1$$

$$2 \int \tan^4 \theta \sec \theta d\theta =$$

$$* \quad c^2 + 1 = \sec^2$$

$$c^2 = \sec^2 \theta - 1$$

$$2 \int (\sec^2 \theta - 1)^2 \tan \theta \sec \theta d\theta \quad u = \sec \theta$$

$$\triangle \quad \sqrt{1-u^2}$$

$$2 \int (u^2 - 1)^2 du = 2 \int u^4 - 2u^2 + 1 du =$$

$$2 \left[\frac{u^5}{5} - \frac{2}{3} u^3 + u \right] = 2 \left[\frac{\sec^5 \theta}{5} - \frac{2}{3} \sec^3 \theta + \sec \theta \right] =$$

$$\text{using} \quad 2 \left[\frac{1}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + (x+1)^{\frac{1}{2}} \right]$$

$$II \quad \int \frac{x^3 + x}{x^3 + 1}$$

$$x^3 + 1 \quad \frac{1}{x^3 + 1}$$

$$= \int \frac{dx}{x} - \int \frac{dx}{x^3 + 1}$$

$$= \int \frac{dx}{x} - \int \frac{dx}{(x+1)(x^2 - x + 1)}$$

II (cont)

$$1 = \frac{A}{x+1} + \frac{bx+c}{x^2-x+1}$$

$$1 = A(x^2-x+1) + (bx+c)(x+1)$$

$$\text{if } x=-1 \quad 1 = 3A \therefore A = \frac{1}{3}$$

$$\text{if } x=0 \quad 1 = A + c \therefore c = \frac{2}{3}$$

$$\text{if } x=+1 \quad 1 = A + 2(b+c)$$

$$1 = \frac{1}{3} + 2b + \frac{4}{3}$$

$$-\frac{2}{3} = 2b \therefore b = -\frac{1}{3}$$

$$= \int \frac{1}{x} - \frac{1}{3} \int \frac{dx}{x+1} + \frac{1}{3} \int \frac{x-2}{x^2-x+1} dx$$

$$x - \frac{1}{3} \ln|x+1| + \frac{1}{3} \int \frac{x-2}{x^2-x+1} dx$$

$$x^2-x+1 = x^2 + x + \frac{1}{4} + \frac{3}{4}$$

$$(x+\frac{1}{2})^2 + \frac{3}{4} = \frac{1}{4} [4(x+\frac{1}{2})^2 + 3]$$

$$\frac{1}{4} [2(x+\frac{1}{2})^2 + 3]$$

$$\frac{1}{4} [(2x+1)^2 + 3]$$

$$3 \frac{1}{4} [(\frac{2x+1}{\sqrt{3}})^2 + 1]$$

$$\frac{1}{3} \int \frac{x-2}{x^2-x+1} dx = \frac{1}{3} \cdot \frac{1}{2} \int \frac{2x-1}{x^2-x+1} dx - \frac{1}{3} \cdot \frac{3}{2} \int \frac{1}{x^2-x+1} dx$$

$$\frac{1}{6} \ln|x^2-x+1| - \frac{1}{2} \int \frac{1}{\frac{3}{4} (\frac{2x+1}{\sqrt{3}})^2 + 1} dx$$

$$- \frac{1}{2} \cdot \frac{1}{\frac{3}{4}} \int \frac{1}{(\frac{2x+1}{\sqrt{3}})^2 + 1} dx$$

$$- \frac{2}{3} \frac{\sqrt{3}}{2} \tan^{-1}(\frac{2x+1}{\sqrt{3}})$$

$$x - \frac{1}{3} \ln|x+1| + \frac{1}{6} \ln|x^2-x+1| - \frac{\sqrt{3}}{3} \tan^{-1}(\frac{2x+1}{\sqrt{3}})$$

Extra.

$$\int_0^1 f(h(t)) dt =$$

$$u = t \quad v = \int h(t) dt = g(t)$$

$$du = dt \quad dv = h(t) dt$$

$$\int g(t) \Big|_0^1 = \int_0^1 g(t) dt$$

$$\int g(t) \Big|_0^1 = f(t) \Big|_0^1$$

$$1(0-0) - (1-0)$$

Copy original problem.

Convince me that you understand the concept!

Chapter 9 Exam

Assume constants of integration are zero.

I

Integrate: $\int \frac{x^2}{\sqrt{1+x}} dx$ using...

(20 pts ea)

a) $u = \sqrt{1+x}$

b) $u = 1+x$

c) $x = \tan^2 \theta$

$\sqrt{x} = \tan \theta$

II

Integrate: $\int \frac{x^3}{x^3+1} dx$

(40 pts)

Extra Credit

5 pts

Given: $f'(t) = g(t)$, $h(t) = g'(t)$, $t \in [0,1]$ Suppose: $f(0) = 0$, $f(1) = 1$, $g(0) = 0$, $g(1) = 0$, $h(0) = 0$, $h(1) = 0$

Find:

$\int_0^1 t h(t) dt$

$$I \int \frac{x^3 dx}{\sqrt{1+x^2}}$$

A) $u = x^2$ $du = 2x dx$
 $v = \sqrt{1+x^2}$

$$x^2 \sqrt{1+x^2} = \int 2x \sqrt{1+x^2} dx$$

$u = 1+x^2$
 $du = 2x dx$

$$= \int u^{\frac{1}{2}} du$$

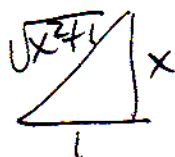
$$x^2 \sqrt{1+x^2} = \frac{2}{3} u^{\frac{3}{2}}$$

$$x^2 \sqrt{1+x^2} = \frac{2}{3} (1+x^2)^{\frac{3}{2}}$$

$$\sqrt{1+x^2} \left(x^2 - \frac{2}{3} - \frac{2}{3} x^2 \right)$$

$$\sqrt{1+x^2} \left[\frac{x^2-2}{3} \right]$$

C) $x = \tan \theta$
 $dx = \sec^2 \theta d\theta$



$$\int \frac{\tan^3 \theta \sec^2 \theta d\theta}{\sec \theta}$$

$$\int \tan^2 \theta \sec \theta \tan \theta d\theta$$

$$s^2 + c^2 = 1$$

$$t^2 + 1 = \sec^2$$

$$(\sec^2 \theta - 1) \sec \theta \tan \theta d\theta$$

$u = \sec \theta$
 $du = \sec \theta \tan \theta d\theta$

$$\int u^2 - 1 du$$

$$\frac{u^3}{3} - u$$

$$\int \frac{x^2 x dx}{\sqrt{1+x^2}}$$

B) $u = x^2 + 1$
 $u+1 = x^2 + 1$
 $du = 2x dx$
 $\frac{1}{2} du = x dx$

$$\frac{1}{2} \int \frac{u du}{(u+1)^{\frac{1}{2}}} = \frac{1}{2} \int \frac{u+1-1}{(u+1)^{\frac{1}{2}}} du$$

$$= \frac{1}{2} \int (u+1)^{\frac{1}{2}} - \frac{1}{2} \int \frac{1}{(u+1)^{\frac{1}{2}}} du$$

$$\frac{1}{2} (u+1)^{\frac{3}{2}} - \sqrt{u+1}$$

$$\frac{1}{3} (u+1)^{\frac{3}{2}} - (u+1)^{\frac{1}{2}}$$

$$(u+1)^{\frac{1}{2}} \left(\frac{1}{3} (u+1) - 1 \right)$$

$$(u+1)^{\frac{1}{2}} \frac{u-2}{3}$$

$$(x^2+1)^{\frac{1}{2}} \left[\frac{x^2-2}{3} \right]$$

$$\frac{\sec^3 \theta}{3} - \sec \theta$$

$$\left(\frac{\sqrt{x^2+1}}{3} \right)^3 - (\sqrt{x^2+1})$$

$$\sqrt{x^2+1} \left(\frac{x^2}{3} + \frac{1}{3} - 1 \right)$$

$$\sqrt{x^2+1} \left(\frac{x^2-2}{3} \right)$$

D) There are no differences to reconcile (after each is simplified)

$$\text{II} \int \frac{\tan^{-1} x}{x^2} dx$$

$$u = \tan^{-1} x \quad v = -\frac{1}{x}$$

$$du = \frac{1}{1+x^2} dx \quad dv = \frac{dx}{x^2}$$

$$-\frac{1}{x} \tan^{-1} x + \int \frac{dx}{x(1+x^2)}$$

$$+ \int \frac{dx}{x} - \int \frac{x dx}{1+x^2}$$

$$-\frac{1}{x} \tan^{-1} x + \ln|x| - \frac{1}{2} \ln(1+x^2)$$

$$\frac{1}{x(1+x^2)} = \frac{A}{x} + \frac{Bx+C}{1+x^2}$$

$$1 = A(1+x^2) + (Bx+C)x$$

$$\text{If } x=0 \quad 1 = A$$

$$\text{If } x=1 \quad 1 = 2 + B + C$$

$$B + C = -1$$

$$\text{If } x=-1 \quad 1 = 2 + B - C$$

$$B - C = -1$$

$$2B = -2$$

$$B = -1$$

$$C = 0$$

$$\text{III} \quad \text{A) } \int_{\sqrt{-3}}^{\sqrt{3}} x^5 \sqrt{x^2+3} dx$$

$$\int_{\sqrt{-3}}^{\sqrt{3}} x^3 \sqrt{x^2+3} x^2 dx$$

$$u^2 = x^2 + 3 \quad -2 = u^2 - 3$$

$$2u du = 2x dx$$

$$\frac{2}{3} u^3 = x^2 dx$$

$$\frac{2}{3} \int_0^1 (u^2-3) u^2 du$$

$$\frac{2}{3} \int_0^1 u^4 - 3u^2 du$$

$$\frac{2}{3} \left(\frac{u^5}{5} - u^3 \right) \Big|_0^1$$

$$\frac{2}{3} \left(\frac{1}{5} - 1 \right)$$

$$\frac{2}{3} \left(-\frac{4}{5} \right)$$

$$-\frac{8}{15}$$

$$\text{A) } \int_{\sqrt{-2}}^{\sqrt{2}} \frac{x^5 \sqrt{x^2+3} dx}{x^3 \sqrt{x^2+3} x^2 dx}$$

$$u = x^2 + 3 \quad u = -3 + 3$$

$$\frac{1}{3} du = x^2 dx \quad u = -2 + 3$$

$$\frac{1}{3} \int_0^1 (u-3) u^{\frac{1}{2}} du$$

$$\frac{1}{3} \int_0^1 u^{\frac{3}{2}} - 3u^{\frac{1}{2}} du$$

$$\frac{1}{3} \left[\frac{2u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3} \right] \Big|_0^1$$

$$\frac{1}{3} \left[\frac{2}{5} - \frac{2}{3} \right]$$

$$\frac{1}{3} \left[\frac{2}{5} - \frac{10}{15} \right]$$

$$\frac{1}{3} \left[-\frac{8}{15} \right]$$

$$-\frac{8}{15}$$

$$\text{B) } \int_0^1 \sqrt{x} \sqrt{1-x^{\frac{3}{2}}} dx$$

$$u = 1 - x^{\frac{3}{2}}$$

$$-du = \frac{3}{2} x^{\frac{1}{2}} dx$$

$$-\frac{2}{3} \int_1^0 u^{\frac{1}{2}} du$$

$$\frac{2}{3} \int_0^1 u^{\frac{1}{2}} du$$

$$\frac{2}{3} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_0^1$$

$$\frac{4}{9}$$

EXTRA

Let $x = \text{steps/sec of local}$

"Steps to top" = $20 + 20x = 32 + 16x$

$x = 3 \text{ steps/sec}$

$20 + 60 = 80 \text{ steps to top}$

Time taken to go to top = $26 \frac{2}{3} \text{ sec}$

$$\text{let } x = \text{Steps/Sec Esc}$$

$$J = \text{Jake's steps/sec}$$

$$20x + 20J = \text{Steps to Top}$$

$$16x + 16 \cdot 2J = \text{Steps to Top}$$

$$20x + 20J = 16x + 32J$$

$$4x = 12J$$

$$x = 3J$$

$$\text{Given } J = 1 \quad \therefore x = 3 \text{ Steps/Sec.}$$

$$20x + 20 \cdot \frac{x}{3} = \text{Steps}$$

$$\frac{60x + 20x}{3}$$

$$\frac{80}{3} x = \text{Steps} \quad \div \quad x \frac{\text{Steps}}{\text{Sec.}}$$

$$\frac{\text{Steps}}{\frac{\text{Steps}}{\text{Sec}}} = \text{Sec}$$

$$\frac{80}{3} \text{ Sec}$$

$$26 \frac{2}{3} \text{ Seconds}$$

It is poor choice because "C" = 0. It won't
 so you can screw up numerator and ~~not~~ make
 a difference.
 Oh - well - leave it alone - Most got it
 set up properly or
 they didn't come
 close.

100	2
90	3
80	3
70	6
60	2
59	3

77.47

105

 38

without the
 38 79.67.

Advanced Placement Calculus

Name Cal Ch 9

Copy original problem.

Per _____

Date 3/24/94Convince *me* that you understand the concept!

Chapter 9 Exam

Assume constants of integration to be zero.

Your method must be very clear to me.

I Given: $f(x) = \int \frac{x^3}{\sqrt{1+x^2}} dx$ (40 pts tot)

a) Find $f(x)$ using $u = x^2$ and using "Parts". (Hint: The "rest of it" is dv .)b) Find $f(x)$ using $u = x^2$ and NOT using "Parts". *hint $u = (u-1) + 1$* c) Find $f(x)$ using $x = \tan \theta$

d) Reconcile any differences among your three previous answers.

II Integrate: $f(x) = \int \frac{\tan^{-1} x}{x^2} dx$ (20 pts)

III Evaluate the definite integral: (20 pts ea)

a) $\int_{\sqrt[3]{-3}}^{\sqrt[3]{2}} x^5 \sqrt{x^3 + 3} dx$

b) $\int_0^1 \sqrt{x} \sqrt{1-x^{3/2}} dx$

Extra Credit ----- 5 pts -----

The escalator in the mall moves up at a rate measured in steps per second. Jake walked up the escalator at the rate of one step per second and reached the top in twenty seconds. Jasper, walking twice as fast as Jake, walked up the escalator in sixteen seconds. The Czarina, being too regal to walk on an escalator, just rode. How long did it take her to get to the top?

$$I \int \frac{x-3}{x^2-6x+8} dx = \frac{a}{x-4} + \frac{b}{x-2}$$

a)

$$x-3 = a(x-2) + b(x-4)$$

$$\text{if } x=2 \quad -1 = -2b \therefore b = \frac{1}{2}$$

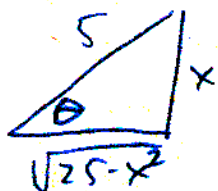
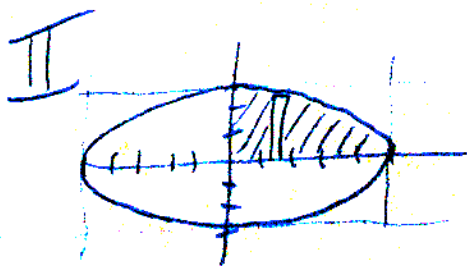
$$\text{if } x=4 \quad 1 = 2a \therefore a = \frac{1}{2}$$

$$= \frac{1}{2} \int \frac{dx}{x-4} + \frac{1}{2} \int \frac{dx}{x-2}$$

$$\frac{1}{2} \ln |(x-4)(x-2)|$$

$$c) \int \frac{x-3}{x^2-6x+8} dx$$

$$\frac{1}{2} \int \frac{2(x-3) dx}{x^2-6x+8} = \frac{1}{2} \ln |x^2-6x+8|$$



$$\text{let } x = 5 \sin \theta$$

$$dx = 5 \cos \theta d\theta$$

$$x=0 \quad \theta=0$$

$$x=5 \quad \theta = \frac{\pi}{2}$$

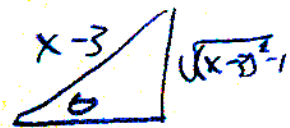
$$\frac{3}{5} \cdot 4 \int_0^{\frac{\pi}{2}} \sqrt{25-x^2} dx$$

$$\frac{12}{5} \int_0^{\frac{\pi}{2}} 5 \cos \theta \cdot 5 \cos \theta d\theta$$

$$60 \int_0^{\frac{\pi}{2}} \cos^2 \theta d\theta$$

$$\frac{60}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2\theta) d\theta$$

$$b) \int \frac{x-3}{x^2-6x+9-1} dx$$



$$x-3 = \sec \theta$$

$$dx = \sec \theta \tan \theta d\theta$$

$$\int \frac{\sec \theta \sec \theta \tan \theta d\theta}{\tan^2 \theta} \quad u = \tan \theta$$

$$du = \sec^2 \theta d\theta$$

$$\int \frac{du}{u} = \ln |u| + C$$

$$\ln |\tan \theta|$$

$$\ln \sqrt{(x-3)^2-1} = \frac{1}{2} \ln (x-4)(x-2)$$

d) all 3 answers are the same.

$$y^2 = (225 - 9x^2) \frac{1}{25}$$

$$y = \sqrt{\frac{225-9x^2}{25}}$$

$$y = \frac{3}{5} \sqrt{25-x^2}$$

$$30 \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{2}}$$

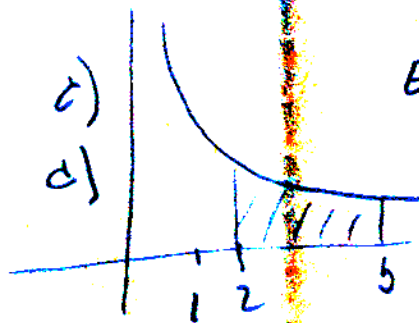
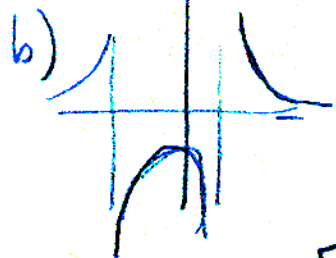
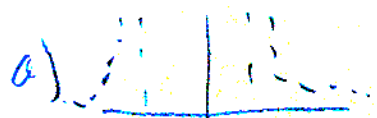
$$30 \left[\frac{\pi}{2} + 0 - 0 \right]$$

$$15\pi = 47.124$$

Calc

$$(12/5) \text{FWINT}(\sqrt{25-x^2}, 0, 5) = 47.124$$

$$\text{III } f(x) = \frac{15}{x^2 + 3x - 4} = \frac{15}{(x+4)(x-1)}$$



$$E) \int_2^7 \frac{15}{(x+4)(x-1)} dx = \frac{a}{x-1} + \frac{b}{x+4}$$

$$15 = a(x+4) + b(x-1)$$

$$15 = 5a \quad \therefore a = 3$$

$$15 = -5b \quad \therefore b = -3$$

$$3 \int \frac{dx}{x-1} - 3 \int \frac{dx}{x+4} = 3 \ln \left| \frac{x-1}{x+4} \right| \Big|_2^7 =$$

$$3 \left[\ln \left| \frac{6}{11} \right| - \ln \left| \frac{1}{6} \right| \right] = 3 \ln \left| \frac{36}{11} \right| \approx 3.557$$

$$F) \text{FNINT}(Y, X, 2, 7) = 3.557 \quad \leftarrow \text{4 gues}$$

$$G) \lim_{b \rightarrow \infty} 3 \ln \left| \frac{x-1}{x+4} \right|_2^b =$$

$$\lim_{b \rightarrow \infty} 3 \left[\ln \left| \frac{b-1}{b+4} \right| - \ln \left| \frac{1}{6} \right| \right]$$

$$3(\ln(1) + \ln 6)$$

$$3 \ln 6.$$

Extra. $\int \ln x \cdot x$

$$u = \ln x \quad du = \frac{1}{x} dx$$

$$du = \frac{1}{x} dx \quad x = x$$

$$x \ln x - \int dx$$

$$x \ln x - x + C$$

Replace I with

something where the integral is given —
they haven't had enough experience
using the integral — Just in
evaluating it.

100	2
90	1
80	6
70	4
60	1
59	1

82.13

105



50

Advanced Placement Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

Assume constants of integration to be zero.

Name

Calc Ch 9

Per

Date

3/20/95

Chapter 9 Exam

I

Given $\int \frac{x-3}{x^2-6x+8} dx$

(30 pts tot)

- Evaluate this integral by first resolving into partial fractions.
- Evaluate this integral by completing the square, followed by trig substitution.
- Evaluate this integral directly, as the integral of the reciprocal function.
- Show that the three answers are equivalent.

II

Find *exactly* the area of the ellipse $9x^2 + 25y^2 = 225$ by integrating using the fundamental theorem. Compare that answer with the approximate answer obtained by using the program RiemannC or the fnInt function of your calculator.

(30 pts)

III

Given: $f(x) = \frac{15}{x^2 + 3x - 4}$

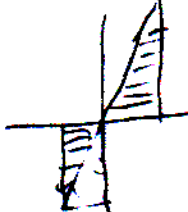
(40 pts tot)

- Set your calculator to "DOT" mode and graph with window: $[-10, 10] [-10, 10]$. Draw your picture.
- Reset your calculator to "CONNECTED" mode. Graph. Draw your picture. The almost vertical lines are due to the fact that your calculator simply connects each point in order -- even across domain restrictions.
- Change the window to $x \in [1, 10]$ $y \in [-1, 5]$. Sketch.
- Indicate the area below $f(x)$ between $x = 2$ and $x = b$ where b stands for a constant greater than 2.
- Compute the *exact* area $\int_2^b f(x) dx$ using the fundamental theorem when $b = 7$.
- Check your answer using RiemannC or the fnInt function of your calculator. (By how much do they differ?)
- The area described in part "e" approaches a finite limit as b approaches infinity. Show that this limit is $3 \ln 6$.

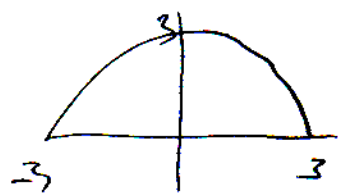
Extra Credit ----- 5 pts -----

Using "parts", derive a formula for $\int \ln ax \, dx$ where a is a constant.

I a) $\int_{-1}^2 3x \, dx$



b) $\int_{-3}^3 \sqrt{9-x^2} \, dx$



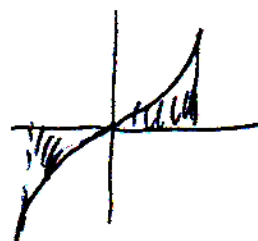
$$\frac{1}{2} \pi \cdot 3^2 = \frac{9\pi}{2}$$

$$\int_{-1}^0 3x \, dx + \int_0^2 3x \, dx$$

$$-\frac{1}{2} \cdot 1 \cdot 3 + 6$$

$$\frac{9}{2}$$

c) $\int_{-1}^1 x^{\frac{1}{3}} \, dx$



The areas
Subtract to
zero.

II a) $\frac{a}{x} + \frac{b}{x^2} + \frac{c}{x^3} + \frac{d}{x^4} + \frac{e}{x^5} + \frac{f(x+G)}{1+x^2} + \frac{h(x+I)}{(1+x^2)^2} + \frac{j(x+K)}{(1+x^2)^3}$ (constant)

b) $\frac{x}{x^5(1+x^2)^3}$

$$\frac{x+2x^3}{x^6(1+x^2)^3}$$

$$\frac{(x^2(1+x^2))^3}{(x^2+x^4)^3}$$

$$\frac{x+2x^3}{(x^2+x^4)^3}$$

c) let $u = x^4 + x^2$
 $du = 4x^3 + 2x$
 $2x(x^2+1)$

$$\frac{1}{2} du = \frac{1+2x^2}{x^5(1+x^2)^3} dx$$

$$\int \frac{x(1+2x^2)}{(x^2+x^4)^3} dx$$

$$\frac{1}{2} \int \frac{du}{u^3}$$

$$\frac{-\frac{1}{2}}{2(x^4+x^2)^2} + C$$

III $f(x) = \frac{\sin x}{1+\cos x}$

a) $\int_0^{\frac{\pi}{2}} f'(x) dx = f(x) \Big|_0^{\frac{\pi}{2}} = \frac{1}{1+0} - 0 = 1$

b) $\int_0^{\frac{\pi}{2}} f(x) dx = -\int_2^1 \frac{du}{u} = \int_1^2 \frac{du}{u} = \ln u \Big|_1^2 = \ln 2 - \ln 1 = \ln 2$

let $u = 1+\cos x$
 $-du = \sin x \, dx$

$$\text{III c) } \int_0^{\frac{\pi}{2}} f(x) f'(x) dx = \left(\frac{f(x)}{2} \right)^2 \bigg|_0^{\frac{\pi}{2}} = \frac{(1-0)^2}{2} = \frac{1}{2}$$

$$\text{D) } \int_0^{\frac{\pi}{2}} \frac{f'(x)}{1+f(x)} dx \quad \text{let } u = f(x) \\ du = f'(x) dx$$

$$\int_0^1 \frac{du}{1+u} = \ln(1+u) = \ln 2 - \ln 1 = \ln 2$$

$$\text{E) } g(x) = \int_0^{f(x)} f(t) dt$$

$$g'(x) = f(f(x)) \cdot f'(x)$$

$$\frac{\sin\left(\frac{\sin x}{1+\cos x}\right)}{1+\cos\left(\frac{\sin x}{1+\cos x}\right)} \cdot \frac{1}{(1+\cos x)}$$

$$\frac{\sin x}{1+\cos x} \cdot \frac{(1+\cos x)\cos x + \sin^2 x}{(1+\cos x)^2} = \frac{\cos x + 1}{(1+\cos x)^2}$$

IV

$$\text{a) } \int \sqrt{9-x^2} dx \quad \begin{array}{c} 3 \\ \swarrow \searrow \\ \sqrt{9-x^2} \end{array}$$

$$\text{let } 3 \sin \theta = x \\ 3 \cos \theta d\theta = dx$$

$$\int 3 \cos \theta \cdot 3 \cos \theta d\theta$$

$$9 \int \cos^2 \theta d\theta$$

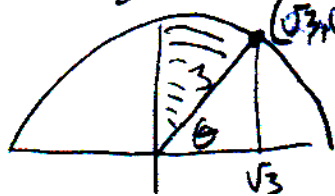
$$\frac{9}{2} \int (1 + \cos 2\theta) d\theta$$

$$\frac{9}{2} \left(\theta + \frac{1}{2} \sin 2\theta \right) + C$$

$$\frac{9}{2} \left(\sin^{-1} \frac{x}{3} + \sin \theta \cos \theta \right) + C$$

$$\frac{9}{2} \left(\sin^{-1} \frac{x}{3} + \frac{x}{3} \cdot \frac{\sqrt{9-x^2}}{3} \right) + C$$

$$\frac{9}{2} \sin^{-1} \frac{x}{3} + \frac{x \sqrt{9-x^2}}{2} + C$$



$$\begin{aligned} \sqrt{9-x^2} &= y \\ 2x^2 &= 9-x^2 \\ 3x^2 &= 9 \\ x &= \pm \sqrt{3} \end{aligned}$$

$$\text{B) } \int x \sqrt{9-x^2} dx$$

$$\text{let } u = 9-x^2 \\ -\frac{1}{2} du = -x dx$$

$$-\frac{1}{2} \int u^{\frac{1}{2}} du$$

$$-\frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} + C$$

$$-\frac{1}{3} (9-x^2)^{\frac{3}{2}} + C$$

$$\text{c) } \int \frac{dx}{\sqrt{x^2-9}} \quad \begin{array}{c} x \\ \swarrow \searrow \\ \sqrt{x^2-9} \end{array}$$

$$\text{let } x = 3 \sec \theta \\ dx = 3 \sec \theta \tan \theta d\theta$$

$$3 \tan \theta = \sqrt{x^2-9}$$

$$\int \frac{3 \sec \theta \tan \theta d\theta}{3 \tan \theta}$$

$$\int \sec \theta d\theta = \ln |\sec \theta + \tan \theta| \\ = \ln \left| \frac{x}{3} + \frac{\sqrt{x^2-9}}{3} \right|$$

Extra

$$\int_0^3 \sqrt{9-x^2} dx + \int_0^3 \sqrt{9-x^2} - x \sqrt{2} dx$$

$$\begin{aligned} \text{Area of Sector } \frac{\theta \cdot r^2}{2} & \quad \tan^{-1} \frac{\sqrt{2}}{1} = \theta \\ \left(\frac{\pi}{2} - \theta \right) \cdot \frac{3^2}{2} & \approx 2.77 \end{aligned}$$

Plus Quarter circle =

$$\frac{1}{4} \pi \cdot 3^2 = \frac{9\pi}{4}$$

total 9.84

$$\frac{9}{4} \left(\pi - \left(\frac{\pi}{2} - \tan^{-1} \sqrt{2} \right) \right)$$

93

106	0
90	3
80	5
70	4
60	1
59	4

43

$$\frac{1273}{17} = 74.9$$

Excellent Exam
No changes.

Advanced Placement Calculus

Copy original problem.

Convince *me* that you understand the concept!

Name

Calc Ch 9

Per

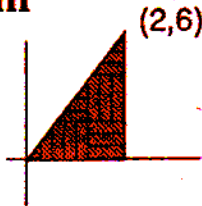
Date

3/98

Chapter 9 Exam

I Evaluate the following definite integrals *without* using calculus.

Example: $\int_0^2 3x \, dx =$



$$= \frac{1}{2} \cdot 2 \cdot 6 = 6$$

(5 pts ea)

a) $\int_{-1}^2 3x \, dx$

b) $\int_{-3}^3 \sqrt{9 - x^2} \, dx$

c) $\int_{-1}^1 x^{\frac{1}{2}} \, dx$

II Given: $\int \frac{1 + 2x^2}{x^5(1 + x^2)^3} \, dx$

(20 pts tot)

a) If you were to set this integral up for partial fractions, how many variables would be required? Write out the set up but do not actually compute the value of any of the constants.

b) Show that the integrand can be re-written as: $\frac{x + 2x^3}{(x^4 + x^2)^3}$

c) Use the substitution $u = x^4 + x^2$ and the part b above to integrate the given integral.

III Given $f(x) = \frac{\sin x}{1 + \cos x}$. Compute each of the following (No calculators)

(7 pts ea)

a) $\int_0^{\frac{\pi}{2}} f'(x) \, dx$

b) $\int_0^{\frac{\pi}{2}} f(x) \, dx$

c) $\int_0^{\frac{\pi}{2}} f(x) \cdot f'(x) \, dx$

d) $\int_0^{\frac{\pi}{2}} \frac{f'(x)}{1 + f(x)} \, dx$

e) $g'(x)$ where $g(x) = \int_0^{f(x)} f(t) \, dt$

IV Integrate:

(10 pts ea)

a) $\int \sqrt{9 - x^2} \, dx$

b) $\int x \sqrt{9 - x^2} \, dx$

c) $\int \frac{1}{\sqrt{x^2 - 9}} \, dx$

Extra Credit ----- 5 pts -----

Given: $f(x) = \begin{cases} 0 & x \leq 0 \\ \sqrt{2}x & x > 0 \end{cases}$

Without using calculus, evaluate: $\int_{-3}^{\sqrt{3}} (\sqrt{9 - x^2} - f(x)) \, dx$

(hint: A calculator may be useful but is not required.)

$f(x) = \int \frac{dx}{x^2 + 4x + 5}$
 $(x+2)^2 + 1$
 $g(x) = \frac{5x}{x^2 + 4x + 5} = \frac{A}{x+5} + \frac{B}{x+1}$
 NOTICE $\rightarrow (x+5)(x+1)$
 $1 = A(x+1) + B(x+5)$
 $x = -5 \quad 1 = -6A \quad A = -\frac{1}{6}$
 $x = -1 \quad 1 = 6B \quad B = \frac{1}{6}$
 $\frac{1}{6} \int \left(-\frac{1}{x+5} + \frac{1}{x+1} \right) dx$
 $\frac{1}{6} \ln \left| \frac{x+1}{x+5} \right| + C$

- b) ~~find~~ ^{using the} ~~explain how~~ D_f and D_g and therefore $f(x) \neq g(x)$
show that $D_f \neq D_g$
- c) ~~find~~ ^{write} $f(x)$ and $g(x)$ without the use of the integral symbol (i.e. without)
- a) $f(2) = ?$ $g(2) = ?$ Use your calculator
- b) Evaluate your answers to part "c" at $x = 2$.
Do your results agree with steps "a"?
explain.

Given $f(x) = \frac{1}{x^2+4x+5}$ $g(x) = \frac{1}{x^2+4x-5}$

a) Obviously, $f(x) \neq g(x)$. Because they have different domains. What are the domains of f and g ?
5 pts

b) $F'(x) = \int f(x) dx$ and $G(x) = \int g(x) dx$

Find $F'(x)$ and $G(x)$.

20 pts.

$$\int \frac{1}{(x^2+4x+5)+1} dx \quad \frac{1}{(x)} = \frac{a}{(x-1)} + \frac{b}{x+5}$$

$$1 = a(x+5) + b(x-1)$$

$$x=1 \quad 1 = 6a \quad \therefore a = \frac{1}{6}$$

$$x=-5 \quad 1 = -6b \quad \therefore b = -\frac{1}{6}$$

$$\int \frac{dx}{x^2+4x+5}$$

$$\frac{1}{6} \int \frac{dx}{x-1} - \frac{1}{6} \int \frac{dx}{x+5}$$

$$\frac{1}{6} \ln|x-1| - \frac{1}{6} \ln|x+5|$$

$$\frac{1}{6} \ln \left| \frac{x-1}{x+5} \right| + C_2$$

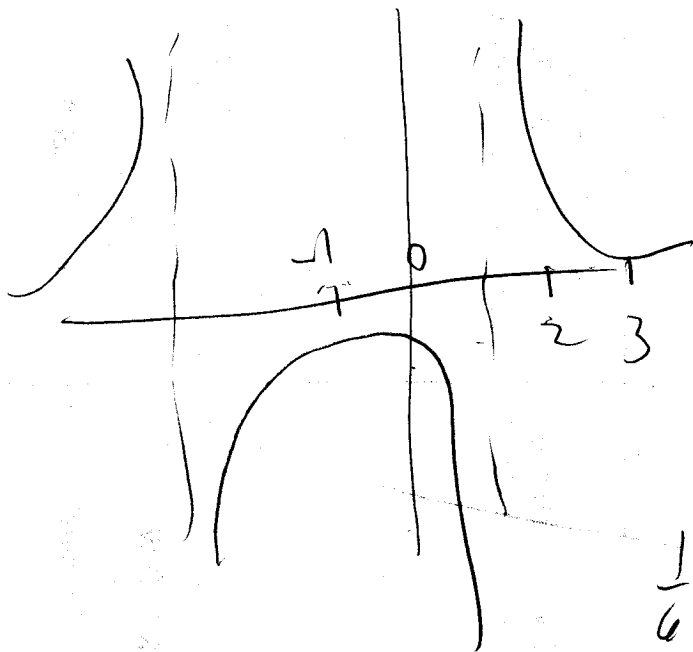
$$F(x) = \tan^{-1}(x+2) + C_1$$

c) $F(z) = \tan^{-1} u + C_1$

$$G(z) = \frac{1}{6} \ln \left| \frac{1}{8} \right| + C_2$$

$$\text{fnInt} \left(\frac{1}{x^4 + 5}, x, 0, 2 \right) = .2187$$

$$\overline{\ln}^{-1}(x+2) \Big|_0^2 = \overline{\ln}^{-1} 4 - \overline{\ln}^{-1} 2 = .2187,$$



$$\frac{1}{x^4 + 5}, x, -1, 0$$

-.153

$$\frac{1}{6} \ln \frac{-2}{4}$$

$$\frac{1}{6} (\ln .5 - \ln .2)$$

$$\frac{1}{6} \ln \left(\frac{5}{2} \right) = (+).153$$

↑

$$\frac{1}{6} \ln \frac{2}{5} = -.153$$

EAs y Partial Fact

$$\int \frac{x+4}{x^3+3x^2-10x} = \frac{a}{x} + \frac{b}{x+5} + \frac{c}{x-2}$$

$$x(x^2+3x-10)$$

$$x(x+5)(x-2)$$

$$x+4 = a(x+5)(x-2) + b(x)(x-2) + c(x)(x+5)$$

$$0 \quad 4 = -10a \quad a = -\frac{2}{5}$$

$$2 \quad 6 = 14c \quad c = \frac{3}{7}$$

$$-5 \quad -1 = -35b \quad b = -\frac{1}{35}$$

$$35$$

$$-\frac{2}{5}(x+5)(x-2) - \frac{1}{35}x(x-2) + \frac{3}{7}x(x+5)$$

$$35x + 140 = -14(x^2+3x-10) - x^2 + 2x + 15x^2 + 75x$$

$$-14x^2 - 42x + 140$$

$$-x^2 + 2x$$

$$+15x^2 + 75x$$

$$37x$$

$$\frac{77}{35}$$

Subject: Re: Even Derivative
 Date: 02/14 8:56 PM
 Received: 02/15 9:24 PM
 From: me@talman1.mscd.edu
 Reply-To: Post to AP listserv, ap-calc@ets.org
 To: Post to AP listserv, ap-calc@ets.org
 CC: wilsonm@westgrand.k12.or.us

Rich Watts wrote:

> I was looking at a problem that showed a picture of $f'(x)$ and then
 > stated that it was even. The problem then asked a question about the
 > integral of $f(x)$ from $-a$ to a . After playing around with it for a
 > while I was pretty confident that the integral would be 0 and that is,
 > in fact, what the answer book said.
 >
 > My question: if a derivative is even, can it always be said that the
 > function will be odd?

Well, let's see. A candidate for $f(x)$ is $\text{Integral}\{0, x\} f'[t] dt$, in notation that I hope is obvious.

We can calculate $f[-x]$ as $\text{Integral}\{0, -x\} f'[t] dt$, and we can evaluate the latter integral by substituting $u = -t$, in which case $du = -dt$. When $t = 0$, we have $u = 0$, and when $t = -x$, we have $u = x$. Thus, by the usual Substitution Theorem for Definite Integrals we must have $f[-x] = \text{Integral}\{0, -x\} f'[t] dt = -\text{Integral}\{0, x\} f'[-t] dt$, and the latter is $-\text{Integral}\{0, x\} f'[t] dt$ because f' is even. Thus $f[-x] = -f[x]$. This insures that $\text{Integral}\{-a, a\} f[t] dt = 0$.

Unfortunately, this isn't the whole story, because the function f given by $f[x] = 3 + \text{Integrate}\{0, x\} f'[t] dt$ is another candidate for f . Here, the same substitution leads to $f[-x] = 3 - f[x] = f[0] - f[x]$. Thus, using the same substitution as above in the second step,

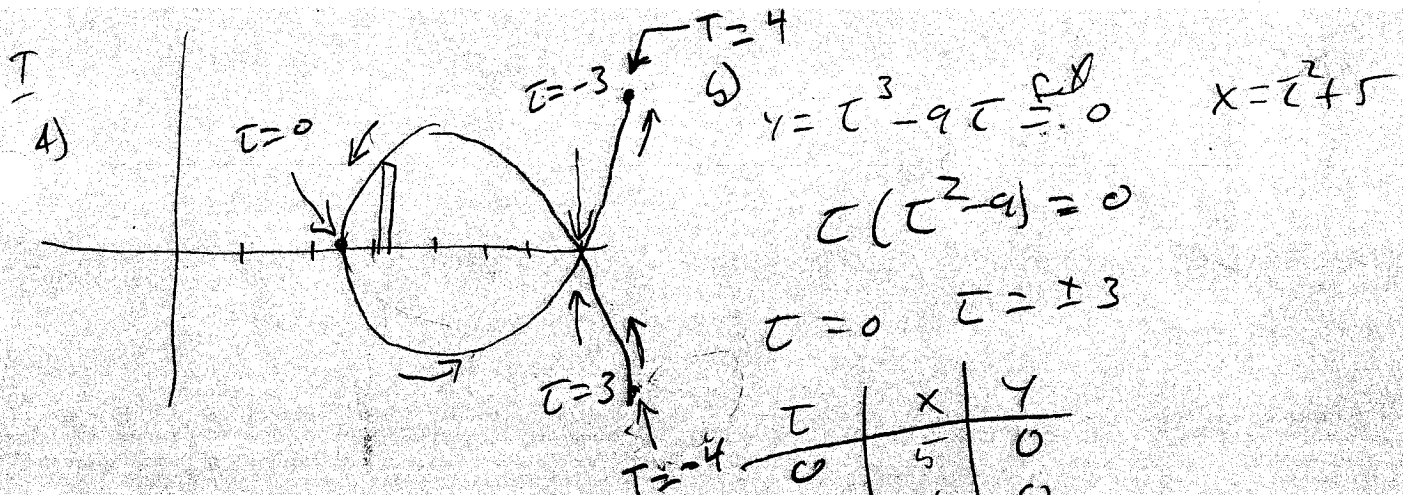
$$\begin{aligned} \text{Integrate}\{-a, a\} f[t] dt &= \text{Integrate}\{-a, 0\} f[t] dt + \text{Integrate}\{0, a\} f[t] dt \\ &= -\text{Integrate}\{a, 0\} f[-u] du + \text{Integrate}\{0, a\} f[t] dt \\ &= \text{Integrate}\{0, a\} (f[0] - f[t]) dt + \text{Integrate}\{0, a\} f[t] dt \\ &= \text{Integrate}\{0, a\} (f[0] - f[t] + f[t]) dt \\ &= \text{Integrate}\{0, a\} f[0] dt \\ &= a f[0]. \end{aligned}$$

Thus, the answer works out to zero only if $f[0] = 0$, in which case (and only in which case) f works out to be odd.

--Lou Talman

"If a derivative is even, ~~can it~~ ~~it can~~ also then the function ~~functions~~ will be odd"

agree or disagree.
 Support your position.



c) $\frac{dy}{dx}$
 $t \in \{0, \pm 3\}$

$y = t^3 - 9t$
 $x = t^2 + 5$

$\frac{dy}{dt} = (3t^2 - 9)$
 $\frac{dy}{dx} = \frac{3t^2 - 9}{2t}$

$\frac{dy}{dx} = \frac{3t^2 - 9}{2t}$
 $t = 0 \neq 3$

$\frac{18}{6} \quad \frac{dy}{dx} = 3 \text{ for } t = 3 \quad \frac{dy}{dx} \text{ does not exist for } t = 0$
 $\frac{18}{-2} \quad \frac{dy}{dx} = -3 \text{ for } t = -3$

$x = t^2 + 5$
 $y = t^3 - 9t$
 $t(t^2 - 9) = 0$
 $t = 0, \pm 3$

d) $\frac{dy}{dx} = 0$
 $\frac{3t^2 - 9}{2t} = 0$

$3t^2 - 9 = 0 \quad t = \pm \sqrt{3}$
 $t^2 = 3 \quad t = \pm \sqrt{3}$
 $t = \pm \sqrt{3}$

e) $2 \int_{t=0}^{t=3} y dx = 2 \int_{t=0}^{t=3} y \cdot \frac{dx}{dt} dt = 2 \int_{t=0}^{t=3} (t^3 - 9t) 2t dt =$

$4 \int_{t=0}^{t=3} (t^4 - 9t^2) dt = 4 \left(\frac{t^5}{5} - 3t^3 \right) \Big|_0^3 = 4 \left(\frac{243}{5} - 81 \right) = \frac{648}{5}$

$\frac{405}{243}$
 $\frac{162}{4}$
 $\frac{648}{5}$

IV f)

	θ	r
1	0	b
2	$\frac{\pi}{6}$	$a\frac{\sqrt{3}}{2} + b\frac{\sqrt{3}}{2}$
3	$\frac{\pi}{4}$	$a\frac{\sqrt{2}}{2} + b\frac{\sqrt{2}}{2}$
4	$\frac{\pi}{3}$	$a\frac{\sqrt{3}}{2} + b$
5	$\frac{\pi}{2}$	a
6	$\frac{2\pi}{3}$	$a\frac{\sqrt{3}}{2} - b\frac{\sqrt{3}}{2}$
7	$\frac{3\pi}{4}$	$a\frac{\sqrt{2}}{2} - b\frac{\sqrt{2}}{2}$
8	$\frac{5\pi}{6}$	$\frac{a}{2} - b\frac{\sqrt{3}}{2}$
9	π	$b = r(0)$
10	$\frac{7\pi}{6}$	$-\frac{a}{2} - b\frac{\sqrt{3}}{2} = -(\frac{a}{2} + b\frac{\sqrt{3}}{2}) = r(\frac{\pi}{6})$

II g) Starting with line
we are repeating the same
points. The significance
of this is that the
entire figure is drawn
when $0 \leq \theta \leq \pi$

II h)

$$\text{area} \int_{\alpha}^{\beta} \frac{r^2}{2} d\theta \rightarrow \frac{1}{2} \int_0^{\pi} (a \sin \theta + b \cos \theta)^2 d\theta$$

$$= \frac{1}{2} \int_0^{\pi} a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta d\theta$$

$$= \frac{1}{2} \int_0^{\pi} \frac{a^2}{2} (1 - \cos 2\theta) + \frac{b^2}{2} (1 + \cos 2\theta) + 2ab \sin \theta \cos \theta d\theta$$

$$= \frac{1}{2} \left[\frac{a^2}{2} \cdot \theta - \frac{a^2}{4} \sin 2\theta + \frac{b^2}{2} \theta + \frac{b^2}{4} \sin 2\theta + \frac{2ab \sin^2 \theta}{2} \right]_0^{\pi}$$

$$= \frac{1}{2} \left[\frac{a^2}{2} \pi - 0 + \frac{b^2}{2} \pi + 0 + 0 - 0 \right] = \frac{\pi}{4} (a^2 + b^2)$$

II i)

$$L(\theta) = \int_{\alpha}^{\theta} \sqrt{r^2 + \dot{r}^2} d\theta$$

$$r = a \sin \theta + b \cos \theta$$

$$r^2 = a^2 \sin^2 \theta + ab \sin \theta \cos \theta + b^2 \cos^2 \theta$$

$$\dot{r} = a \cos \theta - b \sin \theta$$

$$\dot{r}^2 = a^2 \cos^2 \theta - 2ab \sin \theta \cos \theta + b^2 \sin^2 \theta$$

$$= \int_{\alpha}^{\theta} \sqrt{a^2 + b^2} d\theta = \sqrt{a^2 + b^2} \theta \Big|_0^{\pi} = \pi \sqrt{a^2 + b^2}$$

Π_J

area of circle Πr^2

$$\left(x - \frac{b}{2}\right)^2 + \left(y - \frac{a}{2}\right)^2 = \left(\frac{\sqrt{a^2 + b^2}}{2}\right)^2$$

$$\therefore \Pi \left(\frac{a^2 + b^2}{4}\right) \text{ same as } \Pi r^2$$

circumference of circle $2\Pi r$

$$\cancel{2\Pi \frac{\sqrt{a^2 + b^2}}{2}} = \Pi \sqrt{a^2 + b^2} \text{ same as } \Pi r$$

change Equation #2 on calc to Skel
chPT 9 to Read!

$$\frac{4x^2 - 19x + 13}{(x-2)^2(2x-1)} = \frac{1}{x-2} - \frac{3}{(x-2)^2} + \frac{2}{2x-1}$$

method to create "nice" fractions

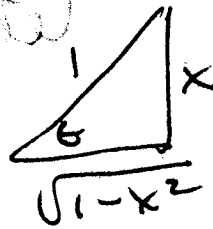
$$\frac{Px^2 + Qx + R}{(x-d)^2(ex+f)} = \frac{a}{x-d} + \frac{b}{(x-d)^2} + \frac{c}{ex+f}$$

choose d, e, f

choose a, b, c

Simplify (R) and P, Q, R will appear.

$$\int_0^1 \frac{x^2 dx}{\sqrt{1-x^2}} = \frac{1}{2} \int_0^{\pi/2} \sin^2 \theta \cos \theta d\theta$$



$$\begin{aligned} \text{Let } x &= \sin \theta & x=0 & \theta=0 \\ dx &= \cos \theta d\theta & x=1 & \theta=\frac{\pi}{2} \end{aligned}$$

$$\int \frac{\sin^2 \theta \cos \theta d\theta}{\cos \theta}$$

$$\int \sin^2 \theta d\theta = \frac{1}{2} \int_0^{\pi/2} (1 - \cos 2\theta) d\theta$$

$$= \frac{1}{2} \left(\theta - \frac{1}{2} \sin 2\theta \right) \Big|_0^{\pi/2}$$

$$= \frac{1}{2} \left[\frac{\pi}{2} - 0 - (0 - 0) \right] = \frac{\pi}{4}$$

from part "a"

$$\frac{\pi}{4} - \frac{1}{2} \cdot \frac{\pi}{4}$$

from break up via "parts"

$$\frac{\pi}{8} \checkmark$$

Jim Mumaugh - Vaca High

From: Jim [teeitup98@sbcglobal.net]
Sent: Sunday, March 02, 2003 8:08 PM
To: Jim Mumaugh - Vaca High
Subject: FW: [ap-calculus] Re: Separable Diff Eq

could have
more used
extra credit
~~calc 1003~~

----- Forwarded Message

> From: "A. Jorge Garcia" <calcpage@lycos.com>
> Organization: Lycos Mail (http://www.mail.lycos.com:80)
> Reply-To: "AP-Calculus" <ap-calculus@lyris.collegeboard.com>
> Date: Fri, 28 Feb 2003 20:22:22 -0500
> To: "AP-Calculus" <ap-calculus@lyris.collegeboard.com>
> Cc: calcpage@lycos.com
> Subject: [ap-calculus] Re: Separable Diff Eq

>>>
> dy/dx=x+2+xy+2y, y(0)=9
> <<
> OK, lets try!

> GCF:
> dy/dx = x(1+y) + 2(1+y)

> GCF:
> dy/dx = (1+y)(2+x)

> VarSep:
> dy/(1+y) = (2+x)dx

> Anti-Differentiate with an arbitrary const of integration:

> ln|1+y| = 2x+x^2/2+A

> Use inverse of ln(x):
> 1+y=Be^(2x+x^2/2)

> Solve for y, General Solution:
> y=Be^(2x+x^2/2)-1

> Use initial condition:
> 9=Be^(0)-1

> Find B=e^A:
> B=10

> Particular Solution:
> y(x)=10e^(2x+x^2/2)-1

> How's that? Looks like you are correct!

> BTW, I'll be giving a talk about this very topic at LIMACON on 3/13 at
SUNY

> Old Westbury. IE: DiffEqus - Graphical, Numerical and Analytical!

Anyone

> else out there going?

> ---

> Regards,

> A. Jorge Garcia

> calcpage@lycos.com

> Teacher & Professor

> Applied Math & CompSci

$$\begin{aligned} \frac{dy}{dx} &= x+2+xy+2y \\ &= x(1+y)+2(1+y) \\ &= (x+2)(1+y) \\ \frac{dy}{1+y} &= (x+2)dx \end{aligned}$$

$$\begin{aligned} \ln|1+y| &= \frac{x^2}{2} + 2x + C \\ 1+y &= A e^{\frac{x^2}{2} + 2x} \\ 10 &= A \\ y &= 10 e^{\frac{x^2}{2} + 2x} - 1 \end{aligned}$$

Math 16b 100 New and Improved Practice Integrals

1. $\int (x + e^x)^2 dx$

3. $\int \ln(1+x) dx$

5. $\int (\cos^3 2x) (\sin^2 2x) dx$

7. $\int \tan x \sec^5 x dx$

9. $\int \frac{x^2 \cos(x^3+7)}{\sin(x^3+7)} dx$

11. $\int x^3 \sqrt{4-x^2} dx$

13. $\int \frac{x^3+1}{x(x-1)^3} dx$

15. $\int \frac{x^3-20x^2-63x-198}{x^2-81} dx$

17. $\int e^x \cos x dx$

19. $\int (x-1) \sqrt{3-x} dx$

21. $\int e^{2x} \sin 3x dx$

23. $\int \sin^3 x \cos^3 x dx$

25. $\int \frac{x}{\sqrt{4-x^2}} dx$

27. $\int \frac{x^5-x^3+1}{x^3+2x^2} dx$

29. $\int \frac{1}{\sqrt{x^3} + \sqrt{x}} dx$

31. $\int e^x \sec^2 e^x dx$

33. $\int x^2 \sin 5x dx$

35. $\int \sin^3 x \sqrt{\cos x} dx$

37. $\int e^x \sqrt{1+e^x} dx$

2. $\int \csc^2 x dx$

4. $\int e^{\sqrt{x}} dx$

6. $\int (\cos^3 x) (\sin x) dx$

8. $\int \tan x \sec^6 x dx$

10. $\int \frac{x}{\sqrt{16-x^2}} dx$

12. $\int \frac{x}{(x^2+1)^2} dx$

14. $\int \frac{1}{x-x^3} dx$

16. $\int \frac{x-1}{(x+2)^5} dx$

18. $\int x e^{x^{\frac{1}{3}}} dx$

20. $\int \frac{\sin x}{2 \cos x + 3} dx$

22. $\int \cos(\ln x) dx$

24. $\int \sec 3x \tan 3x dx$

26. $\int \frac{4-(\ln x)^3}{x} dx$

28. $\int \frac{x^3}{x^3-9x^2+27x-27} dx$

30. $\int \frac{2x+1}{(x+5)^{100}} dx$

32. $\int x \tan x^2 dx$

34. $\int e^2 dx$

36. $\int \sin 3x \cot 3x dx$

38. $\int x(4x^2+25)^{-\frac{1}{2}} dx$

$$39. \int \frac{x^3}{\sqrt{4x^2 + 25}} dx$$

$$41. \int \sec^2 x \tan x dx$$

$$43. \int x \csc^2 x dx$$

$$45. \int x^2 \sqrt{8 - x^3} dx$$

$$47. \int \cos \sqrt{x} dx$$

$$49. \int \frac{e^{3x}}{1 + e^x} dx$$

$$51. \int \frac{x^2 - 4x + 3}{\sqrt{x}} dx$$

$$53. \int \frac{x^3}{\sqrt{16 - x^2}} dx$$

$$55. \int \frac{1 - 2x}{x^2 + 12x + 35} dx$$

$$57. \int (3x + 1)^4 dx$$

$$59. \int \frac{e^{\tan x}}{\cos^2 x} dx$$

$$61. \int \frac{3x^3 - 16x^2 + 29x - 4}{(x + 1)(x - 2)^3} dx$$

$$63. \int \cot^3 x \csc^2 x dx$$

$$65. \int x^3 \sqrt{x^2 - 25} dx$$

$$67. \int \frac{\cot x}{\ln(\sin x)} dx$$

$$69. \int x^2 e^{-4x} dx$$

$$71. \int \frac{35 + 7x}{\sqrt{11 - 10x - x^2}} dx$$

$$40. \int \frac{3x + 2}{x^2 + 10x + 25} dx$$

$$42. \int \sin^2 x \cos^5 x dx$$

$$44. \int (x^3 + 2) \sin x dx$$

$$46. \int x (\ln x)^2 dx$$

$$48. \int x \sqrt{5 - 3x} dx$$

$$50. \int \frac{e^{2x}}{4 - e^{4x}} dx$$

$$52. \int \frac{\cos^3 x}{\sqrt{1 + \sin x}} dx$$

$$54. \int \frac{x}{25 - 9x^2} dx$$

$$56. \int \frac{7}{x^2 - 8x + 15} dx$$

$$58. \int \sin^3 3x dx$$

$$60. \int \frac{x}{\csc 5x^2} dx$$

$$62. \int \frac{x^3}{\sqrt[3]{x^2 + 4}} dx$$

$$64. \int \cot x \csc^5 x dx$$

$$66. \int (1 + \ln x) e^{x \ln x} dx$$

$$68. \int \frac{x e^x + x e^{-x}}{2} dx$$

$$70. \int x^5 \sqrt{x^3 + 1} dx$$

$$72. \int \frac{12x^3 + 7x}{x^4} dx$$

$$73. \int \tan 7x \cos 7x \, dx$$

$$75. \int \frac{4x^2 - 12x - 10}{(x-2)(x^2 - 4x + 3)} \, dx$$

$$77. \int (x^3 + 1) \cos x \, dx$$

$$79. \int \sqrt{1 - \cos^2 x} \, dx$$

$$81. \int x^2 (x+5)^{3/4} \, dx$$

$$83. \int \frac{1}{x(\sqrt{x} + \sqrt[4]{x})} \, dx$$

$$85. \int \frac{\sin x}{\sqrt{1 + \cos x}} \, dx$$

$$87. \int e^x (\sec e^x) (\tan e^x) \, dx$$

$$89. \int \frac{x^2 + x}{(4 - 3x^2 - 2x^3)^4} \, dx$$

$$91. \int \frac{x^3 + 3x - 2}{x^2 - x} \, dx$$

$$93. \int \frac{(x^2 - 2)^2}{x} \, dx$$

$$95. \int (\sqrt{x})^3 \ln x \, dx$$

$$97. \int \frac{x^2}{\sqrt{2x+3}} \, dx$$

$$99. \int x^3 e^{x^2} \, dx$$

$$74. \int e^{1 + \ln 5x} \, dx$$

$$76. \int \frac{x^3}{\sqrt{16 - x^2}} \, dx$$

$$78. \int (x-3)^2 (x+1) \, dx$$

$$80. \int \frac{4x^3 - 15x^2 - 6x + 81}{x^4 - 18x^2 + 81} \, dx$$

$$82. \int x (x^2 + 5)^{3/4} \, dx$$

$$84. \int x \tan x^2 \, dx$$

$$86. \int \frac{4x^2 - 6x + 4}{(x^2 + 4)(x-2)} \, dx$$

$$88. \int \sin^4 x \cos^3 x \, dx$$

$$90. \int \frac{x}{\sqrt{4 + 9x^2}} \, dx$$

$$92. \int \frac{\sin x}{(1 + \cos x)^3} \, dx$$

$$94. \int \cot x \csc^2 x \, dx$$

$$96. \int \frac{x}{\sqrt[3]{x-1}} \, dx$$

$$98. \int \frac{1 - \cos x}{\cot x} \, dx$$

$$100. \int (x+2)^2 (x+1)^{10} \, dx$$

$\int_1^{\sqrt{2}} (x-1)(1-\sqrt{x^2-1}) \, dx$ involves
 Sec & ad Sec³ & substitution of limits